Available online at sjuoz.uoz.edu.krd
journals.uoz.edu.krd
Science Journal of University of Zakho
Vol. 11, No. 2, pp. 185 –189, April-June, 2023
p-ISSN: 2410-7549
e-ISSN: 2414-6943

THE NEW RANK ONE CLASS FOR UNCONSTRAINED PROBLEMS SOLVING

Ahmed Anwer Mustafa*
Mathematics Dep., Faculty of Education, Zakho University, Kurdistan Region – Iraq
ahmed.mustafa@uoz.edu.krd

Received:31, Oct.,2022, /Accepted: 09 Jan..,2023/Published:25, Apr.,2023 *https://doi.org/10.25271/sjuoz.2023.11.2.1049*

ABSTRACT:

One of the most well-known methods for unconstrained problems is the quasi-Newton approach, iterative solutions. The great precision and quick convergence of the quasi-Newton methods are well recognized. In this work, the new algorithm for the symmetric rank one SR1 method is driven.

The strong Wolfe line search criteria define the step length selection. We also proved the new quasi-Newton equation and positive definite matrix theorem. Preliminary computer testing on the set of fourteen unrestricted optimization test functions leads to the conclusion that this new method is more effective and durable than the implementation of classical SR1 method in terms of iterations count and functions.

KEYWORDS: Quasi-Newton, Symmetric Rank One, Positive Definite Matrix, Unconstrained Optimization, Nonlinear Optimization.

INTRODUCTION

There are several initiatives to improve the Hessian matrix's approximation. Zhang J. and Xu Ch. proposed the modified Quasi-Newton condition (Zhang and Xu Ch.,2001). They used both gradient and function-meaning knowledge to achieve a higher-order accuracy in approximating the second curvature of the objective function.

Based on the extended quasi-Newton condition that was modified by authors in (Issam, Basim and Aadil ,2022), the symmetric rank one update ensures that the approach preserves its symmetric and positive definite properties. It also guarantees global and superliner convergence. The following unrestricted optimization issue is taken into consideration:

$$\text{Min.} f(x), \; x \in R^n \qquad (*)$$

where $f : R^n \to R$, n is the number of variables and, is a continuously differentiable function. There are several iterative approaches available to solve the issue (*) One of the most often-used approaches is the quasi-Newton (QN) method (Farzin , Malik and Wah ,2009).

The classic quasi-Newton equation is satisfied by some well-known updates of H, but very few of them have been able to compete numerically with the well-known Cullum and Brayton (SR1) formula (Cullum and Brayton,1979) (Issam, Basim and Aadil A.,2022).

$$H_{k+1} = H_k + \frac{(v_k - H_k y_k)(v_k - H_k y_k)^T}{y_k^T (v_k - H_k y_k)}$$

To get more updates on the values of H_{k+1}, see (Basheer,hl, 2016),(Dennis & Schnabel,1982), (Fletcher,1980),(Gill, P, Murray&Wright,1981),(Mahmood & Farqad ,2017)(Philipp,2013), (Saad & Jaafer ,2022) and (Wah & Malik A. ,2009) iteratively determining a new solution approximation using quasi-Newton techniques.

$$x_{k+1} = x_k + \alpha_k d_k, \; k = 0,1,2, \ldots$$

where x_k is currently the iterative point., $\alpha_k > 0$ is a step length and d_k is the search direction. This direction is calculated by

$$d_k = -H_k g_k$$

Every time an iteration occurs, the matrix H_k is changed to a new Hessian inverse approximation, H_{k+1}, for which the usual QN equation applies.:

$$H_{k+1} y_k = v_k$$

where $v_k = \alpha_k d_k = x_{k+1} - x_k$, $y_k = g_{k+1} - g_k$

The strong Wolfe terms (SWT), which is the line search method is defined as follows:

$$f(x_k + \alpha_k d_k) \le f(x_k) + c_1 \alpha_k g_k^T d_k$$
$$|g(x_k + \alpha_k d_k)^T d_k| \le -c_2 g_k^T d_k$$

where $0 < c_1 < c_2 < 1$

THE NEW METHOD'S DERIVATION AND ITS ALGORITHM

Derivation of New Algorithm

To identify a minimum of many dimensions' nonlinear functions, the method's basic concept is to apply a modified quasi-Newton condition approximation. The modified quasi-Newton condition is as follows.

$$H_{k+1} \bar{y}_k = v_k$$

where $\bar{y}_k = \bar{g}_{k+1} - g_k$ and $\bar{g}_{k+1} = g_k - \psi \frac{g_k{}^T v_k}{v_k{}^T y_k} y_k$ such that $\psi > 0$ and $v_k^T y_k \neq 0$

The correction term in rank one is $\alpha_k z_k z_k^T$ where $\alpha_k \in R$ and $z_k \in R^n$. Therefore, the updated equation is

$$H_{k+1} = H_k + \alpha_k z_k z_k^T \qquad (1)$$

Observe that if H_k is symmetric, then so is H_{k+1} . Our goal now is to determine α_k *and* z_k.

given H_k , $\bar{y}_k$ and v_k so that the required relationship

$$H_{k+1} \bar{y}_k = v_k \qquad (2)$$

is satisfied. In other words, given H_k , $\bar{y}_k$ and v_k we wish to find α_k *and* z_k , to ensure that

$$H_{k+1} \bar{y}_k = (H_k + \alpha_k z_k z_k^T) \bar{y}_k = v_k$$

first, note that $z_k^T \bar{y}_k$ is a scalar. Thus

$$v_k - H_k \bar{y}_k = (\alpha_k z_k^T \bar{y}_k) z_k \qquad (3)$$

and hence $z_k = \frac{v_k - H_k \bar{y}_k}{\alpha_k (z_k^T \bar{y}_k)}$

We can now determine $\alpha_k z_k z_k^T = \frac{(v_k - H_k \bar{y}_k)(v_k - H_k \bar{y}_k)^T}{\alpha_k (z_k^T \bar{y}_k)^2}$

Hence,

$$H_{k+1} = H_k + \frac{(v_k - H_k \bar{y}_k)(v_k - H_k \bar{y}_k)^T}{\alpha_k (z_k^T \bar{y}_k)^2} \qquad (4)$$

* Corresponding author

Now multiply (3) by $\bar{y}_k^T$ to obtain
$\bar{y}_k^T v_k - \bar{y}_k^T H_k \bar{y}_k = \bar{y}_k^T(\alpha_k z_k^T \bar{y}_k) z_k$
Since α_k is scalar and $\bar{y}_k^T z_k = z_k^T \bar{y}_k$, then the above equation becomes

$$\bar{y}_k^T v_k - \bar{y}_k^T H_k \bar{y}_k = \alpha_k (z_k^T \bar{y}_k)^2 \quad (5)$$

By putting equation (5) in equation (4) we have,

$$H_{k+1} = H_k + \frac{(v_k - H_k\bar{y}_k)(v_k - H_k\bar{y}_k)^T}{\bar{y}_k^T(v_k - H_k\bar{y}_k)} \quad (6)$$

Which is the new algorithm of symmetric rank one

2.2 Algorithm of New Method

Step (1): Given $x_0 \in R^n$ an initial vector, $H_0 \in R^{n\times n}$ a symmetric and positive definite matrix, $\psi > 0$, $\epsilon > 0$ a termination scalar. $k = 0$ and compute $f(x_0), g_0$
Step (2): Compute $g_k = \nabla f(x_k)$
Step (3): Compute $d_k = -H_k g_k$.
Step (4): Find $\alpha_k > 0$ satisfying the strong Wolfe condition.
Step (5): Calculate $v_k = \alpha_k d_k$, $x_{k+1} = x_k + v_k$, $g_{k+1} = \nabla f(x_{k+1})$, $y_k = g_{k+1} - g_k$
Step (6): Compute $\bar{y}_k = \bar{g}_{k+1} - g_k$,
$\bar{g}_{k+1} = g_k - \psi \frac{g_k{}^T v_k}{v_k{}^T y_k} y_k$, If $\|g_{k+1}\| \le \epsilon$ stop.
Step (7): $H_{k+1} = H_k + \frac{(v_k - H_k\bar{y}_k)(v_k - H_k\bar{y}_k)^T}{\bar{y}_k^T(v_k - H_k\bar{y}_k)}$
Step (9): If $|g_{k+1}{}^T g_k| \ge 0.2\|g_{k+1}\|^2$ go to step (3)
Step (10): Set k=k+1, and go to step (4).

Theorem I: For the new algorithm applied to the quadratic functions with Hessian matrix $Q = Q^T$, we have

$$H_{k+1}\bar{y}_k = v_k, k \ge 0.$$

Proof: Multiplying both sides of (6) by $\bar{y}_k$ from the left, we find
$H_{k+1}\bar{y}_k = H_k\bar{y}_k + \frac{(v_k - H_k\bar{y}_k)(v_k - H_k\bar{y}_k)^T}{\bar{y}_k^T(v_k - H_k\bar{y}_k)}\bar{y}_k$
Since $\bar{y}_k^T(v_k - H_k\bar{y}_k)$ and $(v_k - H_k\bar{y}_k)^T\bar{y}_k$ are scalars, then
$H_{k+1}\bar{y}_k = H_k\bar{y}_k + v_k - H_k\bar{y}_k$. So, we have $H_{k+1}\bar{y}_k = v_k$

Theorem II: The matrix H_{k+1} produced by the new monotonic approach is positively definite if H_k is positive definite.

Proof: Multiply equation (6) by $\bar{y}_k$ from the right and by $\bar{y}_k^T$ from the left, we get
$\bar{y}_k^T H_{k+1}\bar{y}_k = \bar{y}_k^T H_k\bar{y}_k + \bar{y}_k^T \frac{(v_k - H_k\bar{y}_k)(v_k - H_k\bar{y}_k)^T}{\bar{y}_k^T(v_k - H_k\bar{y}_k)}\bar{y}_k$
$\bar{y}_k^T(v_k - H_k\bar{y}_k)$ and $(v_k - H_k\bar{y}_k)^T\bar{y}_k$ are scalars, so
$\bar{y}_k^T(v_k - H_k\bar{y}_k) = (v_k - H_k\bar{y}_k)^T\bar{y}_k$ So, we have
$\bar{y}_k^T H_{k+1}\bar{y}_k = \bar{y}_k^T H_k\bar{y}_k + \bar{y}_k^T(v_k - H_k\bar{y}_k) = \bar{y}_k^T H_k\bar{y}_k + \bar{y}_k^T v_k - \bar{y}_k^T H_k\bar{y}_k$
$\Rightarrow \bar{y}_k^T H_{k+1}\bar{y}_k = v_k{}^T\bar{y}_k$

$$v_k{}^T\bar{y}_k = v_k{}^T(\bar{g}_{k+1} - g_k) = v_k{}^T\left(g_k - \psi\frac{g_k{}^T v_k}{v_k{}^T y_k}y_k - g_k\right)$$

$$\Rightarrow v_k{}^T\bar{y}_k = v_k{}^T g_k - \psi\frac{g_k{}^T v_k}{v_k{}^T y_k}v_k{}^T y_k - v_k{}^T g_k$$

Since $v_k = \alpha_k d_k$ and $d_k = -H_k g_k$, we have
$v_k{}^T\bar{y}_k = -\psi g_k{}^T v_k = -\psi g_k{}^T\alpha_k d_k = -\psi\alpha_k g_k{}^T(-H_k g_k) \Rightarrow v_k{}^T y_k = \psi\alpha_k g_k{}^T H_k g_k$
Since H_k, ψ and α_k are positive then H_{k+1} is also positive which makes the proof complete.

ILLUSTRATIONS AND TABLES

This part focused on testing the implementation of the new method. The recent upgrade of SR1 and standard SR1 are evaluated in this study . Well-known nonlinear problems (classical test function) with various functions are used in the comparison testing for $5 \le N \le 2000$. All programs are created in FORTRAN 95, and the terminating condition is present in every case $\|g_{k+1}\| \le 10^{-5}$
and restart using Powell's condition $|g_k{}^T g_{k+1}| \ge 0.2\|g_{k+1}\|^2$.The line search routine was a cubic interpolation that uses function and gradient values.
The numbers of iterations, ISN Sand, and FSN are particularly mentioned in the findings in Tables (3. I) and (3. II). These tables of experimental findings demonstrate that the new algorithm outperforms the traditional approach of SR1 in terms of the number of iterations ISN and the number of functions FSN.

Table (3. I) Comparison of the Standard SR1 and the New Algorithm

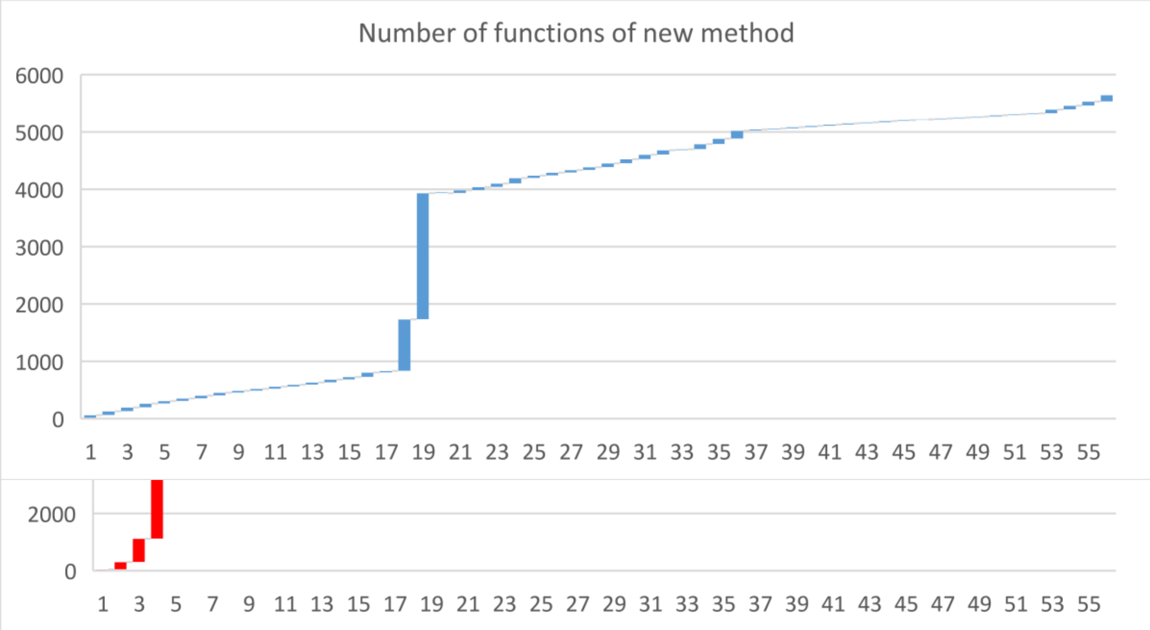
No. of Check	Check Function	N	Standard Formula (SR1)		New Formula	
			ISN	FSN	ISN	FSN
1	Wood	5	40	194	21	61
		100	266	918	24	68
		500	815	3067	24	67
		2000	2223	6937	24	68
2	Cubic	5	19	67	15	44
		100	49	144	16	46
		500	59	181	18	50
		2000	72	224	18	50
3	Rosen	5	35	112	12	34
		100	328	1246	12	34
		500	991	4196	12	34
		2000	1699	51420	12	34
4	Powell	5	19	64	15	41
		100	71	211	18	48
		500	52	168	19	49
		2000	41	120	28	75
5	Sum	5	10	47	9	32
		100	439	1892	224	897
		500	1329	5323	541	2198
		2000	FF	FF	FF	FF

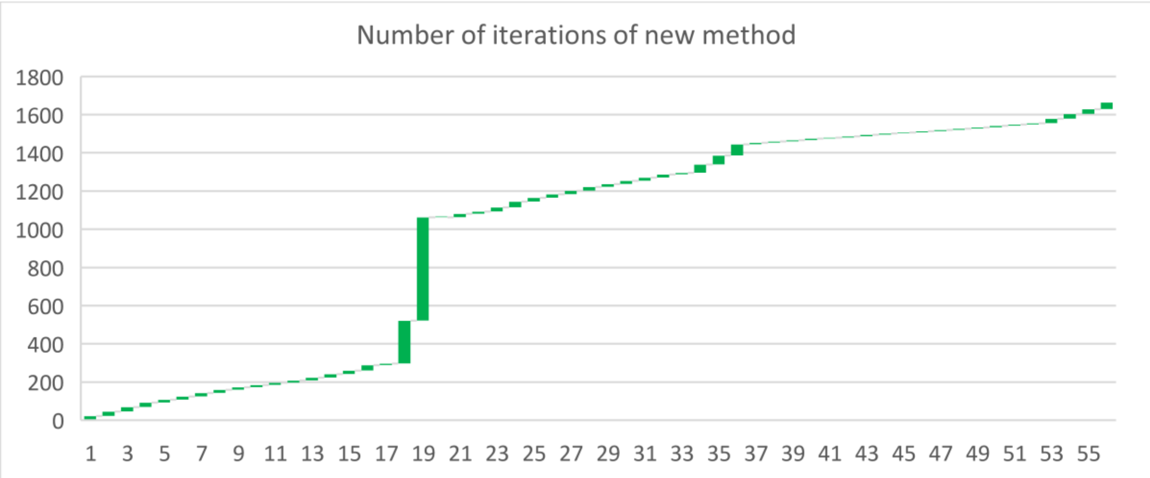
6	Nondiagonal	5 100 500 2000	22 42 42 50	74 175 283 236	18 12 22 31	53 57 61 94
7	G-Helical	5 100 500 2000	20 FF FF FF	62 FF FF FF	19 19 19 19	47 47 47 47
8	G-Central	5 100 500 2000	21 22 23 23	254 260 266 266	16 16 17 17	71 71 78 78
9	Wolfe	5 100 500 2000	8 72 82 182	19 157 178 772	9 43 46 60	19 88 95 138
10	Beal	5 100 500 2000	9 10 10 11	27 27 27 29	7 7 7 8	21 21 21 23
11	Fred	5 100 500 2000	7 8 10 10	24 28 29 29	6 7 7 7	19 21 21 21
12	Resip	5 100 500 2000	6 6 6 6	19 19 19 19	6 6 7 7	14 14 16 16
13	Shallow	5 100 500 2000	8 8 8 10	28 28 28 32	7 7 7 7	19 19 19 19
14	Miele	5 100 500 2000	29 35 35 46	97 142 122 150	24 25 25 35	65 69 69 112
Totals			4661	63452	1468	5084

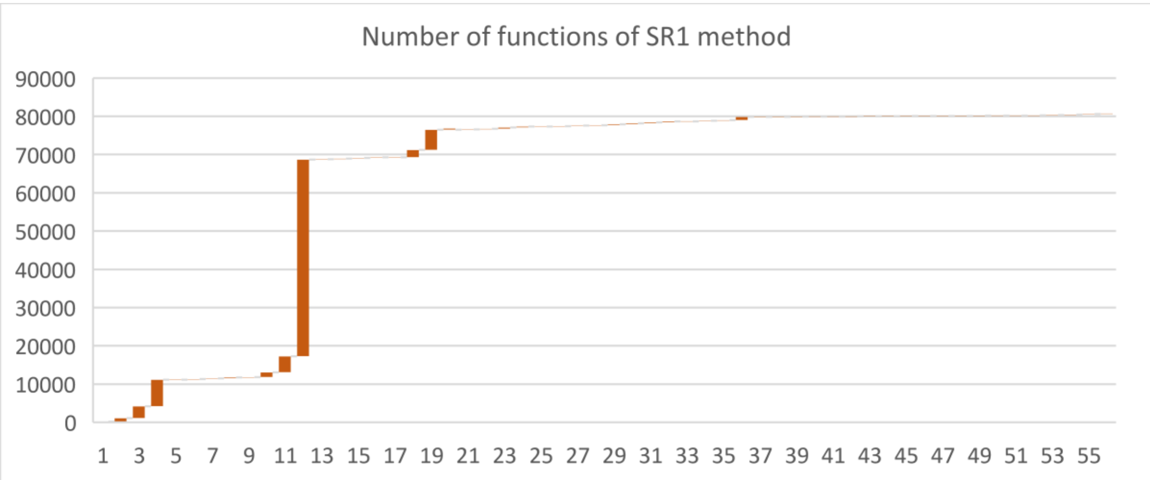
Note, FF means failure and we mean that the function did not work in the standard algorithm and worked in the new algorithm and when calculating the rate of improvement, we took the multipler value the function that did not work in the standard algorithm.

Table (3. II) A comparison of the new method's and the old algorithm's rates of improvement (SR1)

Tools	SR1-Method	New Method
ISN	100%	31.4954%
FSN	100%	8.0124%







Figures (3. I) Performing algorithms for the ISN and the FSN

Table (3. I), Table (3. II), and Fig. (3. I) compare the rate of improvement of the new algorithm to the traditional technologies. (SR1), The numerical results of the new algorithm are better than the standard algorithm since we observe that (ISN), and (FSN) of the standard algorithm (SR1) are about 100%, This means the new algorithm has improved on the standard algorithm (SR1) prorate (68.5046%) in (ISN) and prorate (91.9876%) in (FSN). The new algorithm has generally improved prorate (80.2461%) compared to conventional algorithms (SR1).

CONCLUSION

In this study, we introduced a brand-new symmetric rank one (SR1) approach with a few unique characteristics, such as the quasi-Newtonian requirement and the positive definite property. According to numerical results, this new algorithm outperforms the conventional symmetric rank one.

Future updates and adjustments to the DFP and BFGS nonlinear unconstrained optimization techniques can be proposed in the same manner.

ABBREVIATIONS: QN Quasi Newton SR1 symmetric rank one, Min minimum, g_k=$\nabla f(x_k)$ gradients, SWT strong Wolfe Terms, ISN iterations number, FSN functions number.

REFERENCES

Basheer M. S., Khalil K. A., Zeyad M. A. (2016). Partial Davidon, Fletcher and Powell (DFP) of quasi newton method for unconstrained optimization. Tikrit Journal of Pure Science, 21(6),180-186.

Cullum, j., and Brayton, R. K. (1979) Some Remarks on the Symmetric Rank-One Update, Journal of Optimization Theory and Applications. 27(4), 493-519.

Dennis, J.E., Schnabel, R.B. (1982). Numerical Methods for Nonlinear Equations and Unconstrained Optimization Prentice Hall, Englewood Cliffs.

Farzin M., Malik A. H. and Wah J. L. (2009). Memoryless Modified Symmetric Rank-One Method for Large-Scale Unconstrained Optimization, American Journal of Applied Sciences,6(12),2054-2059.

Fletcher, R. (1980). Practical Methods of Optimization. Wiley, New York.

Gill, P.E., Murray, W., Wright, M.H. (1981). Practical Optimization. Academic, London.

Issam A. R. M., Basim A. H., Aadil A. (2022). New Self-scaling Quasi-Newton methods for unconstrained optimization. International Journal of Mathematics and Computer Science,17(3), 1061-1077.

Mahmood S. S. and Farqad H. (2017). On Extended Symmetric Rank One Update for Unconstrained Optimization," J. of Education. Special issued, 206-220.

Philipp H. Martin K.(2013). Quasi-Newton Methods: A New Direction. Journal of Machine Learning Research, 14, 843-865..

Saad Sh. M. and Jaafer H. E. (2022). New Class of Rank 1 Update for Solving Unconstrained Optimization Problem. Iraqi Journal of Science, 63(2), 683-689.

Wah J. L., Malik A. H.(2009).A restarting approach for the symmetric rank one update for unconstrained optimization. Comput. Optim. Appl.,42, 327–334.

Zhang J. and Xu Ch.(2001). Properties and Numerical Performance of Modified quasi–Newton Methods Equations. Elsevier, J. of Comp. and App. Math,137, 269-278.