$(i,j) - S_c -$ Continuous and $(\mathrm{i},\mathrm{j}) - \theta S_C -$ continuous Functions

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ABSTRACT

In this paper, we introduce new types of continuity in bitopological spaces called $(i,j) - S_C -$ continuous and $(i,j) - \theta S_C -$ continuous. We discuss the relationship between these types of continuity and other known types of continuous functions.

***Keywords*:** $(i,j) - S_C -$ continuous, $(i,j) - \theta S_C -$ continuous, Semi – continuous.

1. Introduction

In [Kelley, 1963] Kelley was first introduced the concept of bitopological spaces, where X is a nonempty set and τ_1, τ_2 are topologies on X. Throughout this paper, (X, τ_1, τ_2) is a bitopological space, and if $A \subseteq X$, then $i - Int(A)$ and $i - Cl(A)$ denote respectively the interior and closure of A with respect to the topology τ_i on X.

A subset A of a space (X, τ) is called preopen [Mashhour, 1982] (resp., semi-open [Levine, 1963], regular open [Stone, 1937]) if $A \subseteq Int(Cl(A))$ (resp., $A \subseteq Cl(Int(A))$, $A = Int(Cl(A))$).

The complement of a preopen (resp., semi-open, regular open) set is said to be preclosed (resp., semi-closed, regular closed).

The intersection of all preclosed (resp., semi – closed) sets of X containing A is called preclosure [El-Deeb, 1983] (resp., semi – closure [Crossely, 1971]).

The union of all preopen (resp., semi – open) sets of X contained in A is called preinterior (resp., semi - interior).

A subset A of a space X is called $\theta -$ semi – open [Joseph, 1980], if for each $x \in A$, there exists a semi – open set G such that $x \in G \subseteq Cl(G) \subseteq A$. A point $x \in X$ is said to be in semi – θ – closure [Di Maio, 1987] (resp., $\theta -$ semi – closure [Joseph, 1980]) of a subset A of X denoted by $sCl_\theta(A)$ (resp.,$\theta sCl(A)$), if $A \cap sCl(U) \neq \emptyset$ (resp., $A \cap Cl(U) \neq \emptyset$), for each semi – open set U of X containing x. A subset A of a space X is said to be $\theta -$ semi closed, if $A = \theta sCl(A)$.

In [Levine, 1963] Levine defined a function $f: X \longrightarrow Y$ to be semi – continuous if $f^{-1}(V)$ is semi – open set in X for every open set V of Y. A function $f: X \longrightarrow Y$ is said to be $\theta s -$ continuous in sense of khalaf and Easif [Khalaf, 1999] (resp., RC- continuous [Dontchev, 1999]) if the inverse image of each open subset of Y is $\theta -$ semi – open (resp., regular closed) in X. A function $f: X \longrightarrow Y$ is said to be $\theta -$ irresolute in sense of park and park [Park, 1995] (resp., $\theta -$ irresolute in sense of Kheder and Noiri [Kheder, 1986], weakly $\theta -$ irresolute [Ganster, 1988]), if for each $x \in X$ and each semi–open set V of Y containing $f(x)$, there exists a semi–open set U of X containing x such that $f(sCl(U)) \subseteq sCl(V)$ (resp., $f(Cl(U)) \subseteq Cl(V)$, $f(U) \subseteq Cl(V)$).

In [Joseph,1980] Joseph and Kwack introduced a function $f: X \longrightarrow Y$ to be $(\theta, s) -$ continuous, if for each $x \in X$ and each semi – open set V of Y containing $f(x)$, there exists an open set U of X containing x such that $f(U) \subseteq Cl(V)$. A function $f: X \longrightarrow Y$ is called $R -$ map [Carnahan, 1973], if the inverse image of each regular open subset of Y is regular open in X. In the present paper we introduce and investigate the concept of $(i,j) - S_C -$ continuous and $(i,j) - \theta S_C -$ continuous functions.

2. Preliminaries

We give the following definition and results which are used in next sections.

Definition 2.1 A space X is regular if for each $x \in X$ and each open set G containing x, there exists an open set H such that $x \in H \subseteq Cl(H) \subseteq G$.

Definition 2.2 [Ganster, 1990] A space X is said to be strongly s–regular space, if for each closed set A and any point $x \notin X \backslash A$, there exists a regular closed set F of X such that $x \in F$ and $F \cap A = \emptyset$.

Definition 2.3 [Arya, 1982] A space X is said to be s–Urysohn, if for each two distinct points x, y in X, there exist two semi – open sets U, V such that $x \in U, y \in V, y \notin U$ and $x \notin V$ and $Cl(U) \cap Cl(V) = \emptyset$.

Definition 2.4 [Stone, 1937] A space X is said to be extermally disconnected, if $Cl(G)$ is open for every open set G of X.

Definition 2.5 [Hardi, 2012] A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(i,j) - S_C -$ open, if A is $j -$ semi $-$ open and for all $x \in A$, there exist an $i -$ closed set F such that $x \in F \subseteq A$. A subset B of X is called $(i,j) - S_C -$ closed if and only if B^C is $(i,j) - S_C -$ open. The family of $(i,j) - S_C -$ open (resp., $(i,j) - S_C -$ closed) subset of X is denoted by $(i,j) - S_C O(X)$ (resp., $(i,j) - S_C C(X)$).

Theorem 2.6: Let A be a subset of a topological space (X, τ), then:

1- If $A \in SO(X)$, then $pCl(A) = Cl(A)$. [Dontchev, 2000]

2- If $A \in \tau$, then $Cl_\theta(A) = Cl(A)$. [Velicko, 1986].

3- $A \in PO(X)$ if and only if $sCl(A) = Int(Cl(A))$. [Jankovich, 1985].

4- If A is open set in a space X, then $sCl(A) = Int(Cl(A))$. [Di Maio, 1987].

Theorem 2.7 [Hardi, 2012] A subset A of a bitopological space (X, τ_1, τ_2) is an $(i,j) - S_C -$ open set if and only if for each $x \in A$, there exists an $(i,j) - S_C -$ open set B such that $x \in B \subseteq A$.

Theorem 2.8 [Hardi, 2012] Let (X, τ_1, τ_2) be a bitopological space. If (X, τ_i) is a $T_1 -$ space, then $(i,j) - S_C O(X) = j - SO(X)$.

Theorem 2.9 [Hardi, 2012] Let (X, τ_1, τ_2) be a bitopological space. If (X, τ_i) is a regular space, then $\tau_j \subseteq (i,j) - S_C O(X)$.

Theorem 2.10 [Donchev, 19988] The intersection of preopen set A and regular closed set B is regular closed in the preopen set A.

Theorem 2.11[Hardi, 2012] Let Y be a subspace of a bitopological space (X, τ_1, τ_2) and $A \subseteq Y$. If $A \in (i,j) - S_C O(Y)$ and $Y \in j - RC(X) \cap i - RC(X)$, then $A \in (i,j) - S_C O(X)$.

Proposition 2.12 [Hardi, 2012] Let X_1, X_2 be two bitopological spaces. If $A \in (i,j) - S_C O(X_1)$ and $B \in (i,j) - S_C O(X_2)$, then $A \times B \in (i,j) - S_C O(X_1 \times X_2)$.

3. $(i,j) - S_C -$ Continuous Function

Definition 3.1: A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called $(i,j) - S_C -$ continuous at a point $x \in X$, if for each $j -$ open V of Y containing $f(x)$, there exists an $(i,j) - S_C -$ open set U of X containing x such that $f(U) \subseteq V$.

If f is $(i,j) - S_C -$ continuous at every point x of X, then it is called $(i,j) - S_C -$ continuous.

Proposition 3.2: A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is $(i,j) - S_C -$ continuous if and only if the inverse image of every $j -$open set in Y is an $(i,j) - S_C -$ open set in X.

Proof (→): Let f be an $(i,j) - S_C -$ continuous function and let V be any $j -$ open set in Y. If $f^{-1}(V) = \phi$ implies $f^{-1}(V)$ is an $(i,j) - S_C -$ open set in X, if $f^{-1}(V) \neq \phi$, then there exists $x \in f^{-1}(V)$ implies $f(x) \in V$. Since f is $(i,j) - S_C -$ continuous, there exists an $(i,j) - S_C -$ open set U in X containing x such that $f(U) \subseteq V$, this implies that $x \in U \subseteq f^{-1}(V)$. Therefore, by Theorem 2.7. , $f^{-1}(V)$ is $(i,j) - S_C -$ open set in X.

(←): Let V be any $j -$ open set in Y, then by hypothesis, $f^{-1}(V)$ is an $(i,j) - S_C -$ open set in X containing x and $f(f^{-1}(V)) \subseteq V$. Therefore, f is $(i,j) - S_C -$ continuous.

It is evident that every $(i,j) - S_C -$ continuous function is $j -$ semi $-$ continuous, but the converse may not be true.

Example 3.3 : Let $X = \{a, b, c\}$, $\tau_1 = \{X, \{a\}, \phi\}$ and $\tau_2 = \{X, \{a, c\}, \phi\}$, then the identity function $f: (X, \tau_1, \tau_2) \longrightarrow (X, \tau_1, \tau_2)$ is j-semi-continuous but it is not $(i,j) - S_C -$ continuous because $\{a, c\}$ is not $(i,j) - S_C$-open set.

Proposition 3.4: A function $f: (X, \tau_1, \tau_2) \longrightarrow (Y, \sigma_1, \sigma_2)$ is called $(i,j) - S_C -$ continuous if and only f is $j -$ semi $-$ continuous and for each $x \in X$ and each $j -$ open set V of Y containing $f(x)$, there exists an $i -$ closed set F of X containing x such that $f(F) \subseteq V$.

Proof: (→): Let f be an $(i,j) - S_C -$ continuous and let $x \in X$ and V be any $j -$open set of Y containing $f(x)$. By hypothesis, there exists an $(i,j) - S_C -$ open set U of X containing x such that $f(U) \subseteq V$. Since, U is an $(i,j) - S_C -$ open set, then for each $x \in U$, there exists an $i -$ closed set F of X such that $x \in F \subseteq U$. Therefore, we have $f(F) \subseteq V$ and $(i,j) - S_C -$ continuous always implies $j -$ semi $-$ continuous.

(←): Let V be any $j -$ open set of Y, we have to show that $f^{-1}(V)$ is an $(i,j) - S_C -$ open set in X. Since, f is $j -$semi–continuous, then $f^{-1}(V)$ is $j -$ semi $-$ open set in X. Let $x \in f^{-1}(V)$, then $f(x) \in V$. By hypothesis there exists an $i -$ closed set F of X containing x

such that $f(F) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is an $(i,j)-S_C-$ open set in X. Hence, by Proposition 3.2, f is $(i,j)-S_C-$ continuous.

Proposition 3.5: For a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following statements are equivalent:

1- f is $(i,j)-S_C-$ continuous.

2- $f^{-1}(V)$ is an $(i,j)-S_C-$ open set in X, for each $j-$ open set V in Y.

3- $f^{-1}(F)$ is an $(i,j)-S_C-$ closed set in X, for each $j-$ closed set F in Y.

4- $f\big((i,j)-S_CCl(A)\big) \subseteq j-Cl\big(f(A)\big)$, for each subset A of X.

5- $(i,j)-S_CCl\big(f^{-1}(B)\big) \subseteq f^{-1}\big(j-$ *ClB*, for each subset *B* of *Y*.

6- $f^{-1}\big(j-Int(B)\big) \subseteq (i,j)-S_CInt\big(f^{-1}(B)\big)$, for each subset B of Y.

7- $j-Int\big(f(A)\big) \subseteq f\big((i,j)-S_CInt(A)\big)$, for each subset A of X.

Proof: 1 ⟹ 2: Follows from **Proposition 3.2.**

2 ⟹ 3: Let F be any $j-$ closed set of Y. Then $Y-F$ is $j-$ open in Y. By (2), $f^{-1}(Y-$ *F=X−f−1F* is an *i, j−SC−* open set in *X*. Hence, $f^{-1}(F)$ is $(i,j)-S_C-$ closed in X.

3 ⟹ 4: Let A be any subset of X, then $f(A) \subseteq j-Cl\big(f(A)\big)$ and $j-Cl\big(f(A)\big)$ is $j-$ closed set in Y. Hence $A \subseteq f^{-1}\Big(j-Cl\big(f(A)\big)\Big)$, by (3) we have, $f^{-1}\Big(j-Cl\big(f(A)\big)\Big)$ is an $(i,j)-S_C-$ closed set in X. Therefore, $(i,j)-S_CCl(A) \subseteq f^{-1}\Big(j-Cl\big(f(A)\big)\Big)$.

Therefore, $f\big((i,j)-S_CCl(A)\big) \subseteq j-Cl\big(f(A)\big)$.

4 ⟹ 5: Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X, by (4), we have, $f\Big((i,j)-$ *SCClf−1B⊆j−Clff−1B=j−ClB.* Hence, $(i,j)-S_CCl\big(f^{-1}(B)\big) \subseteq f^{-1}\big(j-Cl(B)\big)$.

5 ⟹ 6: Let B be any subset of Y. Then if we apply (5) to $Y-B$, we obtain

$$(i,j)-S_CCl\big(f^{-1}(Y-B)\big) \subseteq f^{-1}\big(j-Cl(Y-B)\big)$$

if and only if $(i,j)-S_CCl\big(X-f^{-1}(B)\big) \subseteq f^{-1}(Y-(j-Int(B)))$

if and only if $X-\Big((i,j)-S_CInt\big(f^{-1}(B)\big)\Big) \subseteq f^{-1}\Big(Y-\big(j-Int(B)\big)\Big)$

if and only if $f^{-1}\big(j-Int(B)\big) \subseteq (i,j)-S_CInt\big(f^{-1}(B)\big)$. Therefore,

$f^{-1}\big(j-Int(B)\big) \subseteq (i,j)-S_CInt\big(f^{-1}(B)\big)$.

6 ⟹ 7: Let A be any subset of X. Then $f(A)$ is a subset of Y. By (6) we have, $f^{-1}\Big(j-$ *IntfA⊆i, j−SCIntf−1fA=i, j−SCIntA.*

Therefore, $j-Int\big(f(A)\big) \subseteq f\big((i,j)-$ *SCIntA.*

7 ⟹ 1: Let $x \in X$ and let V be any $j-$ open set of Y containing $f(x)$. Then $x \in f^{-1}(V)$ and $f^{-1}(V)$ is a subset of X, by (7), we have $j-Int\Big(f\big(f^{-1}(V)\big)\Big) \subseteq f\Big((i,j)-$ *SCIntf−1V.* Then

$j-Int(V) \subseteq f\Big((i,j)-S_CInt\big(f^{-1}(V)\big)\Big)$.

Since V is $j-$ open set, then $V \subseteq f\Big((i,j)-S_CInt\big(f^{-1}(V)\big)\Big)$, implies that $f^{-1}(V) \subseteq (i,j)-S_CInt\big(f^{-1}(V)\big)$. Therefore, $f^{-1}(V)$ is an $(i,j)-S_C-$ open set in X which containing x and $f\big(f^{-1}(V)\big) \subseteq V$. Hence, f is $(i,j)-S_C-$ continuous.

1- Proposition 3.6: If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-S_C-$ continuous, then $(i,j)-S_CCl\big(f^{-1}(V)\big) \subseteq f^{-1}\big(j-Cl_\theta(V)\big)$, for each $j-$ open set V in Y.

Proof: Let V be any $j-$ open set in Y. Suppose that $x \notin f^{-1}\big(j-Cl_\theta(V)\big)$, then $f(x) \notin j-Cl_\theta(V)$, so there exists a $j-$ open set G containing $f(x)$ such that $j-Cl(G) \cap V = \phi$ implies $G \cap V = \phi$. Since f is $(i,j)-S_C-$ continuous, then there exists an $(i,j)-S_C-$ open set U containing x such that $f(U) \subseteq G$. Therefore, $f(U) \cap V = \phi$ and $U \cap f^{-1}(V) = \phi$. This shows that $x \notin (i,j)-S_CCl\big(f^{-1}(V)\big)$. Hence

$(i,j)-S_CCl\big(f^{-1}(V)\big) \subseteq f^{-1}\big(j-Cl_\theta(V)\big)$

Proposition 3.7: Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be a function, and β be any basis for σ_j in Y. Then f is $(i,j)-S_C-$ continuous if and only if for each $B \in \beta$, $f^{-1}(B)$ is an $(i,j)-S_C-$ open subset of X.

Proof: (→): Suppose that f is $(i,j)-S_C-$ continuous. Since $B \in \beta$ is $j-$ open set in Y and f $(i,j)-S_C-$ continuous, then by Proposition 3.2, $f^{-1}(B)$ is $(i,j)-S_C-$ open subset of X.

(←): Let V be any $j-$ open subset of Y. Then $V = \cup\{B_i : i \in I\}$, where every B_i is a member of β and I is a suitable index set. It follows that

$f^{-1}(V) = f^{-1}(\cup\{B_i : i \in I\}) = \cup f^{-1}(\{B_i : i \in I\})$. Since, $f^{-1}(B_i)$ is an $(i,j)-S_C-$ open subset of X, for each $i \in I$. Then $f^{-1}(V)$ is a union of a family of $(i,j)-S_C-$ open sets of X. Hence $f^{-1}(V)$ is $(i,j)-S_C-$ open set of X. Therefore, by Proposition 3.2, f is $(i,j)-S_C-$ continuous.

Corollary 3.8: If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $j-$ continuous and (X,τ_i) is regular space, then f is $(i,j)-S_C-$ continuous.

Proof: Follows from Theorem 2.9.

Proposition 3.9: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be an $(i,j)-S_C-$ continuous and Y is $j-$ open subset of a bitopological space Z, then $f:X\to Z$ is $(i,j)-S_C-$ continuous.

Proof: Let V be any $j-$ open set in Z. Then, $V\cap Y$ is $j-$ open set in Y, since $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(i,j)-S_C-$ continuous, by Proposition 3.2, $f^{-1}(V\cap Y)$ is $(i,j)-S_C-$ open set in X. But $f(x)\in Y$ for each $x\in X$ and thus $f^{-1}(V)=f^{-1}(V\cap Y)$ is an $(i,j)-S_C-$ open set in X. Therefore, by Proposition 3.2, $f:X\to Z$ is $(i,j)-S_C-$ continuous.

4. $(i,j)-\theta S_C-$ Continuous Functions

Definition 4.1: A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is called $(i,j)-\theta S_C-$ continuous at a point $x\in X$, if for $j-$ semi – open set V of Y containing $f(x)$, there exists an $(i,j)-S_C-$ open set U of X containing x such that $f(U)\subseteq j-Cl(V)$.

If f is $(i,j)-\theta S_C-$ continuous at every point x of X, then it is called $(i,j)-\theta S_C-$ continuous.

Proposition 4.2: A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ is called $(i,j)-\theta S_C-$ continuous if and only if for each $x\in X$ and each $j-$ regular closed set F of Y containing $f(x)$, there exists an $(i,j)-S_C-$ open set U of X containing x such that $f(U)\subseteq F$.

Proof: ($\rightarrow$): Let V be a $j-$ semi – open set in Y, so $j-Cl(V)=F$ is $j-$ regular closed, then by hypothesis, there exists an $(i,j)-S_C-$ open set U of X containing x such that $f(U)\subseteq F=j-Cl(V)$. Hence, f is $(i,j)-\theta S_C-$ *continuous.*

($\leftarrow$): *Suppose that* f *is* $(i,j)-\theta S_C-$ *continuous, therefore for each* $j-$ *semi – open set* V *of* Y *containing* $f(x)$, *there exists an* $(i,j)-S_C-$ *open set* U *of* X *containing* x *such that* $f(U)\subseteq j-Cl(V)$, *put* $F=j-Cl(V)$ *which is a* $j-$ *regular closed set, this completes the proof.*

Proposition 4.3: A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ *is* $(i,j)-\theta S_C-$ *continuous if and only if the inverse image of every* $j-$*regular closed set in* Y *is* $(i,j)-S_C-$ *open set in* X.

Proof: ($\rightarrow$): Let f be an $(i,j)-$*continuous function and let* V *be any* $j-$ *regular closed set in* Y. *If* $f^{-1}(V)=\phi$ *implies* $f^{-1}(V)$ *is an* $(i,j)-S_C-$ *open set in* X, *if* $f^{-1}(V)\neq\phi$, *then there exists* $x\in f^{-1}(V)$ *implies* $f(x)\in V$. *Since* f *is* $(i,j)-\theta S_C-$ *continuous, then by Proposition 4.2, there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq j-ClV=V$, *this implies that* $x\in U\subseteq f^{-1}(V)$. *Therefore, by Theorem 2.7.,* $f^{-1}(V)$ *is* $(i,j)-S_C-$ *open set in* X.

($\leftarrow$): *Let* V *be any* $j-$ *regular closed set in* Y *, then by hypothesis,* $f^{-1}(V)$ *is an* $(i,j)-S_C-$ *open set in* X *containing* x *and* $f\left(f^{-1}(V)\right)\subseteq j-ClV$. *Therefore,* f *is* $(i,j)-\theta S_C-$ *continuous.*

The functions $(i,j)-\theta S_C-$ *continuous and* $(i,j)-S_C-$ *continuous are incomparable as it is shown in the following examples:*

Example 4.4: Let $=\{a,b,c\}$, $\tau_1=\{X,\{a\},\phi\}$ *and* $\tau_2=\{X,\{b,c\},\phi\}$ *, then the identity function* $f:(X,\tau_1,\tau_2)\longrightarrow(X,\tau_1,\tau_2)$ *is* $(i,j)-\theta S_C-$*continuous but it is not* $(i,j)-S_C-$ *continuous because* $\{a,c\}$ *is not* $(i,j)-S_C$*-open set.*

Example 4.5: Let $=\{a,b,c,d\}$, $\tau_1=\{X,\{a\},\{b\},\{a,b\},\{a,c,d\},\phi\}$ *and* $\tau_2=\{X,\{a\},\{b\},\{a,b\},\{a,b,c\},\phi\}$, *then we define a function* $f:(X,\tau_1,\tau_2)\longrightarrow(X,\tau_1,\tau_2)$ *as follows:* $f(a)=f(c)=b, f(b)=a$ *and* $f(d)=d$, *so* f *is* $(i,j)-S_C-$*continuous but it is not* $(i,j)-\theta S_C-$ *continuous because* $\{a,c,d\}$ *is j-regular closed set but its inverse is* $\{a,d\}$ *which is not* $(i,j)-S_C$*-open .*

Proposition 4.6: A function $f:(X,\tau_1,\tau_2)\longrightarrow(Y,\sigma_1,\sigma_2)$ *is* $(i,j)-\theta S_C-$ *continuous if and only if for each* $x\in X$ *and each and each* $j-\theta-$ *semi – open set* V *of* Y *containing* $f(x)$, *there exists an* $(i,j)-S_C-$ *open set* U *of* X *containing* x *such that* $f(U)\subseteq V$.

Proof: ($\rightarrow$): *Let* $x\in X$ *and* V *be any* $j-$ *semi – open set of* Y *containing* $f(x)$, *so* $j-Cl(V)$ *is* $j-\theta-$ *semi – open of* Y *containing* $f(x)$. *By hypothesis, there exists an* $(i,j)-S_C-$ *open set* U *of* X *containing* x *such that* $f(U)\subseteq j-Cl(V)$. *This shows that* f *is* $(i,j)-\theta S_C-$ *continuous.*

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($\leftarrow$): *Follows from the fact that* $j-Cl(V)\in j-\theta SO(X)$, *for each* $V\in j-SO(X)$.

Proposition 4.7: For a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$*, the following statements are equivalent:*

f *is* $(i,j)-\theta S_C-$ *continuous.*

For each $x\in X$ *and each* $j-$ *semi*$-$ *open set* V *of* Y *containing* $f(x)$*, there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq j-Cl(V)$*.*

For each $x\in X$ *and each* $j-$ *semi*$-$ *open set* V *of* Y *containing* $f(x)$*, there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq j-pCl(V)$*.*

For each $x\in X$ *and each* $j-$ *regular closed set* F *of* Y *containing* $f(x)$*, there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq F$*.*

For each $x\in X$ *and each* $j-\theta-$ *semi*$-$ *open set* V *of* Y *containing* $f(x)$*, there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq V$*.*

Proof: **1** $\Longrightarrow$ **2**: *Obvious.*

2 $\Longrightarrow$ **3**: *Since* $j-Cl(V)=j-pCl(V)$*, for each* $j-$*semi*$-$ *open set* V *of* Y

3 $\Longrightarrow$ **4**: *Since* $j-Cl(V)\in j-RC(Y)$*, for every* $j-$ *semi*$-$ *open set* V *of* Y *.*

4 $\Longrightarrow$ **5**: *Let* $x\in X$ *and* V *be any* $j-\theta-$ *semi*$-$ *open set of* Y *containing* $f(x)$*. Then for each* $f(x)\in V$*, there exists* $j-$ *regular closed set* F *containing* $f(x)$ *such that* $F\subseteq V$*. By (4), there exists an* $(i,j)-S_C-$ *open set* U *in* X *containing* x *such that* $f(U)\subseteq F\subseteq V$*. This completes the proof.*

5 $\Longrightarrow$ **1**: *It is proved in Proposition 4.6.*

Proposition 4.8: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ *be a* $j-$ *weakly* $\theta-$ *irresolute function and for each* $x\in X$ *and each* $j-\theta-$ *semi*$-$ *open set* V *of* Y *containing* $f(x)$*, there exists an* $i-$ *closed set* F *in* X *containing* X *such that* $f(F)\subseteq V$*, then* f *is* $(i,j)-\theta S_C-$ *continuous.*

Proof: Let V *be any* $j-\theta-$ *semi*$-$ *open set of* Y*. To show* $f^{-1}(V)$ *is an* $(i,j)-S_C-$ *open set in* X*. Since* f *is* $j-$ *weakly* $\theta-$ *irresolute, then* $f^{-1}(V)$ *is* $j-$ *semi*$-$ *open set in* X*. Let* $x\in f^{-1}(V)$ *implies* $f(x)\in V$*. By hypothesis, there exists an* $i-$ *closed set* F *of* X *containing* x *such that* $f(F)\subseteq V$*. This implies that* $x\in F\subseteq f^{-1}(V)$*. Therefore,* $f^{-1}(V)$ *is an* $(i,j)-S_C-$ *open set in* X *and* $f\left(f^{-1}(V)\right)\subseteq V$*. By Proposition 4.6,* f *is* $(i,j)-\theta S_C-$ *continuous.*

Proposition 4.9: For a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$*, the following statements are equivalent:*

f *is* $(i,j)-\theta S_C-$ *continuous.*

$f^{-1}\left(j-Cl(V)\right)$ *is an* $(i,j)-S_C-$*open set in* X*, for each* $j-$ *semi*$-$ *open set* V *in* Y*.*

$f^{-1}\left(j-Int(F)\right)$ *is an* $(i,j)-S_C-$ *closed set in* X*, for each* $j-$ *semi*$-$ *closed set* F *in* Y*.*

$f^{-1}(V)$ *is an* $(i,j)-S_C-$ *closed set in* X*, for each* $j-$ *regular closed set* V *of* Y*.*

$f^{-1}(F)$ *is an* $(i,j)-S_C-$ *open set in* X*, for each* $j-$ *regular closed set* F *of* Y*.*

$f^{-1}(V)$ *is an* $(i,j)-S_C-$ *open set in* X*, for each* $j-\theta-$ *semi*$-$ *open set* V *of* Y*.*

$f^{-1}(F)$ *is an* $(i,j)-S_C-$ *closed set in* X*, for each* $j-\theta-$ *semi*$-$ *closed set* F *of* Y*.*

$f\left((i,j)-S_CCl(A)\right)\subseteq j-\theta sCl\left(f(A)\right)$*, for each subset* A *of* X*.*

$(i,j)-S_CCl\left(f^{-1}(B)\right)\subseteq f^{-1}\left(j-\theta sCl(B)\right)$*, for each subset* B *of* Y*.*

$f^{-1}\left(j-\theta sInt(B)\right)\subseteq(i,j)-S_CInt\left(f^{-1}(B)\right)$*, for each subset* B *of* Y*.*

$j-\theta sInt\left(f(A)\right)\subseteq f\left((i,j)-S_CInt(A)\right)$*, for each subset* A *of* X*.*

Proof: **1** $\Longrightarrow$ **2**: *Let* V *be any* $j-$ *semi*$-$ *open set in* Y *and let* $x\in f^{-1}\left(j-Cl(V)\right)$*. Then* $f(x)\in j-Cl(V)$ *and* $j-Cl(V)$ *is* $j-$ *regular closed set in* Y*. Since* f *is* $(i,j)-\theta S_C-$ *continuous, then by Proposition 4.3, there exists an* $(i,j)-S_C-$ *open set* U *of* X *containing* x *such that* $f(U)\subseteq j-Cl(V)$*, implies that* $x\in U\subseteq f^{-1}\left(j-Cl(V)\right)$*. Therefore, by Theorem 2.7,* $f^{-1}\left(j-Cl(V)\right)$ *is an* $(i,j)-S_C-$ *open set in* X*.*

2 $\Longrightarrow$ **3**: *Let* F *be* $j-$ *semi*$-$ *closed set in* Y*, then* $Y-F$ *is* $j-$ *semi*$-$ *open set in* Y*. By (2),* $f^{-1}\left(j-Cl(Y-F)\right)$ *is an* $(i,j)-S_C-$ *open set in* X *and* $f^{-1}\left(j-Cl(Y-F)\right)=f^{-1}\left(Y-\right.$ *j−IntF=X−f−1j−IntF is an i, j−SC− open set in* X*. Hence,* $f^{-1}\left(j-Int(F)\right)$ *is an* $(i,j)-S_C-$ *closed set in* X*.*

3 $\Longrightarrow$ **4**: *Let* V *be any* $j-$ *regular open in* Y*, then* V *is* $j-$ *semi*$-$ *closed in* Y *and* $j-Int(V)=V$*. By (3),* $f^{-1}\left(j-Int(V)\right)=f^{-1}(V)$ *is an* $(i,j)-S_C-$ *closed set in* X*.*

4 $\Longrightarrow$ **5**: *Let* F *be any* $j-$ *regular closed set of* Y*, then* $Y-F$ *is* $j-$ *regular open set of* Y*. By (4),* $f^{-1}(Y-F)$ *is an* $(i,j)-S_C-$ *closed set in* X *and* $f^{-1}(Y-F)=X-f^{-1}(F)$*. Therefore,* $f^{-1}(F)$ *is an* $(i,j)-S_C-$ *open set in* X*.*

5 ⟹ 6: *This follows from the fact that any $\theta -$ semi – open set is a union of regular closed sets.*

6 ⟹ 7: *Similar to the proof of* **4 ⟹ 5** .

7 ⟹ 8: *Let A be any subset of X and $y \notin j-\theta sCl(f(A))$. Then there exists $V \in j-SO(Y)$ containing y such that $f(A) \cap j-Cl(V) = \phi$. Since,*

$j-Cl(V)$ is $j-\theta-$ semi – open set, so $f^{-1}(j-Cl(V))$ is an $(i,j)-S_C-$ open set in X and $A \cap f^{-1}(j-Cl(V)) = \phi$. Therefore,

$(i,j)-S_CCl(A) \cap f^{-1}(j-Cl(V)) = \phi$ *and*

$f((i,j)-S_CCl(A)) \cap j-Cl(V) = \phi$.

Consequently, we obtain

$y \notin f((i,j)-S_CCl(A))$. *Hence,* $f((i,j)-$ *SCClA⊆j–θsClfA.*

8 ⟹ 9: *Let B be any subset of Y. Then $f^{-1}(B)$ is a subset of X. By (8),*

$f\left((i,j)-S_CCl(f^{-1}(B))\right) \subseteq$

$j-\theta sCl\left(f(f^{-1}(B))\right) = j-\theta sCl(B)$.

Therefore, $(i,j)-S_CCl(f^{-1}(B)) \subseteq$

$f^{-1}(j-\theta sCl(B))$.

9 ⟹ 10: *Let B be any subset of Y, then $Y-B$ also subset of Y. By (9),*

$(i,j)-S_CCl(f^{-1}(Y-B))$

$\subseteq f^{-1}(j-\theta sCl(Y-B))$

$\Leftrightarrow (i,j)-S_CCl(X-f^{-1}(B))$

$\subseteq f^{-1}(Y-(j-\theta sInt(B)))$

$\Leftrightarrow X-\left((i,j)-S_CInt(f^{-1}(B))\right)$

$\subseteq X-f^{-1}(j-\theta sInt(B))$

$\Leftrightarrow f^{-1}(j-\theta sInt(B)) \subseteq$

$(i,j)-S_CInt(f^{-1}(B))$.

Therefore, $f^{-1}(j-\theta sInt(B)) \subseteq (i,j)-$
$S_CInt(f^{-1}(B))$.

10 ⟹ 11: *Let A be a subset of X, then $f(A)$ is a subset of Y. By (10),*

$f^{-1}\left(j-\theta sInt(f(A))\right) \subseteq$

$(i,j)-S_CInt\left(f^{-1}(f(A))\right) = (i,j)-S_CInt(A)$.

Therefore, $j-\theta sInt(f(A)) \subseteq f((i,j)-$
SCIntA.

11 ⟹ 1: *Let $x \in X$ and V be any $j-$ semi – open set of Y containing $f(x)$, then $x \in f^{-1}(j-Cl(V))$ and $f^{-1}(j-Cl(V))$ is a subset of X. By (11),*

$j-\theta sInt\left(f\left(f^{-1}(j-Cl(V))\right)\right) \subseteq$

$f\left((i,j)-S_CInt\left(f^{-1}(j-Cl(V))\right)\right)$. *Then,*

$j-\theta sInt(j-Cl(V)) = j-Cl(V) \subseteq$

$f\left((i,j)-S_CInt\left(f^{-1}(j-Cl(V))\right)\right)$

So, $j-Cl(V) \subseteq f\left((i,j)-S_CInt\left(f^{-1}(j-\right.\right.$
ClV, implies that

$f^{-1}(j-Cl(V)) \subseteq (i,j)-S_CInt\left(f^{-1}(j-\right.$
ClV. Therefore,

$f^{-1}(j-Cl(V))$ is an $(i,j)-S_C-$ open set in X which contain x and

$f\left(f^{-1}(j-Cl(V))\right) \subseteq j-Cl(V)$. Hence, f is $(i,j)-\theta S_C-$ continuous.

Proposition 4.10: For a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following statements are equivalent:

f is $(i,j)-\theta S_C-$ continuous.

$(i,j)-S_CCl(f^{-1}(V)) \subseteq f^{-1}\left(j-\right.$
Intj–ClV, for each j– preopen set V of Y.

$f^{-1}\left(j-Cl(j-Int(F))\right) \subseteq (i,j)-$
$S_CInt(f^{-1}(F))$, for each $j-$ preclosed set F of Y.

Proof: **1 ⟹ 2**: *Let V be any $j-$ preopen set in Y, then $V \subseteq j-Int(j-Cl(V))$ and $j-Int(j-Cl(V))$ is $j-$ regular open set in Y. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.9 part (4),* $f^{-1}\left(j-\right.$
Intj–ClV is an i, j–SC– closed set in X.

Hence, we obtain $(i,j)-S_CCl(f^{-1}(V)) \subseteq$
$f^{-1}\left(j-Int(j-Cl(V))\right)$.

2 ⟹ 3: *Let F be any $j-$ preclosed set in Y, then $Y-F$ is $j-$preopen of Y and by (2),*

$(i,j)-S_CCl(f^{-1}(Y-F)) \subseteq f^{-1}\left(j-\right.$
Intj–ClY–F

If and only if $X-\left((i,j)-S_CInt(f^{-1}(F))\right) \subseteq$
$f^{-1}\left(Y-\left(j-Cl(j-Int(F))\right)\right)$

If and only if $X-\left((i,j)-S_CInt(f^{-1}(F))\right) \subseteq$
$X-f^{-1}\left(j-Cl(j-Int(F))\right)$

Therefore, $f^{-1}\left(j-Cl(j-Int(F))\right) \subseteq (i,j)-$
$S_CInt(f^{-1}(F))$.

3 ⟹ 1: *Let V be any $j-$ regular open set of Y. Then $X-V$ is $j-$ preclosed set of Y, by (3), we have $f^{-1}\left(j-Cl\left(j-Int(X-V)\right)\right) \subseteq (i,j)-S_C Int\left(f^{-1}(X-V)\right)$ if and only if $(i,j)-S_C Cl\left(f^{-1}(V)\right) \subseteq f^{-1}\left(j-Int(j-Cl(V))\right) = f^{-1}(V)$. Therefore, $f^{-1}(V)$ is $(i,j)-S_C-$ closed set in X. So by Proposition 4.8 part (4), f is $(i,j)-\theta S_C-$ continuous.*

Corollary 4.10: For a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following statements are equivalent:
f is $(i,j)-\theta S_C-$ continuous.
$(i,j)-S_C Cl\left(f^{-1}(V)\right) \subseteq f^{-1}\left(j-sCl(V)\right)$, for each $j-$ preopen set V of Y.
$f^{-1}\left(j-sInt(F)\right) \subseteq (i,j)-S_C Int\left(f^{-1}(F)\right)$, for each $j-$ preclosed set F of Y.
Proof: Follows from Proposition 4.9 and the fact that $j-sCl(V) = j-Int\left(j-Cl(V)\right)$, for each $j-$ preopen set V of Y. [Jankovich, 1985].
Proposition 4.11: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous if and only if $f^{-1}(V) \subseteq (i,j)-S_C Int\left(f^{-1}(j-Cl(V))\right)$, for each $j-$ semi$-$ open set V in Y.

Proof: (→): Let V be any $j-$ semi$-$ open set in Y. Then $V \subseteq j-Cl(V)$ and $j-Cl(V)$ is $j-$ regular closed set in Y. Since f is $(i,j)-\theta S_C-$ continuous, so by Proposition 4.8 part (5), $f^{-1}\left(j-Cl(V)\right)$ is an $(i,j)-S_C-$ open set in X. Hence, $f^{-1}(V) \subseteq f^{-1}\left(j-Cl(V)\right) = (i,j)-S_C Int\left(f^{-1}(j-Cl(V))\right)$.

(←): Let V be any $j-$ regular closed set in Y, then V is $j-$ semi $-$ open set of Y. By hypothesis, we have
$f^{-1}(V) \subseteq (i,j)-S_C Int\left(f^{-1}\left(j-Cl(V)\right)\right) = (i,j)-S_C Int\left(f^{-1}(V)\right)$.
Therefore, $f^{-1}(V)$ is an $(i,j)-S_C-$ open set in X. Hence by Proposition 4.8, f is $(i,j)-\theta S_C-$ continuous.
From Proposition 4.11 and Theorem 2.6, we get the following corollaries:
Corollary 4.12: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous if and only if $f^{-1}(V) \subseteq (i,j)-S_C Int\left(f^{-1}(j-pCl(V))\right)$, for each $j-$ semi$-$ open set V of Y.

Corollary 4.13: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous if and only if $(i,j)-S_C Cl\left(f^{-1}\left(j-Int(F)\right)\right) \subseteq f^{-1}(F)$, for each $j-$ semi$-$ closed set F of Y.
Corollary 4.15: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous if and only if $(i,j)-S_C Cl\left(f^{-1}\left(j-pInt(F)\right)\right) \subseteq f^{-1}(F)$, for each $j-$ semi$-$ closed set F of Y.
Proposition 4.16: If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous, then $f^{-1}(V) \subseteq (i,j)-S_C Int\left(f^{-1}\left(j-Int(j-Cl(V))\right)\right)$, for each $j-$ preopen set V of Y.

Proof: Let V be any $j-$ preopen set in Y, then $V \subseteq j-Int\left(j-Cl(V)\right)$ and $j-Int\left(j-Cl(V)\right)$ is $j-$ regular open set in Y. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.8 Part (4), $f^{-1}\left(j-Int(j-Cl(V))\right)$ is an $(i,j)-S_C-$ open set in X. Therefore,

$f^{-1}(V) \subseteq f^{-1}\left(j-Int\left(j-Cl(V)\right)\right) = (i,j)-S_C Int\left(f^{-1}\left(j-Int\left(j-Cl(V)\right)\right)\right)$.
From Proposition 4.16 and Theorem 2.6, we obtain the following corollaries:
Corollary 4.17: If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous, then $f^{-1}\left(j-Cl\left(j-Int(F)\right)\right) \subseteq (i,j)-S_C Int\ (f^{-1}(F))$, for each $j-$ preclosed set F of Y.
Corollary 4.18: If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous, then $f^{-1}\left(j-sInt(F)\right) \subseteq (i,j)-S_C Int\ (f^{-1}(F))$, for each $j-$ preclosed set F of Y.
Proposition 4.19: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is not $(i,j)-\theta S_C-$ continuous at the point x of X if and only if $x \in (i,j)-S_C Bd\left(f^{-1}(F)\right)$ for some $j-$ regular closed subsets F of Y containing $f(x)$.
Proof: (→): If f is not $(i,j)-\theta S_C-$ continuous at $x \in X$, then there exists a $j-$ regular closed set F containing $f(x)$ such that for every $(i,j)-S_C-$ open set U of X containing x, $f(U) \cap (Y-F) \neq \phi$. This means that for every $(i,j)-S_C-$ open set U of X containing x, we have $U \cap \left(X-f^{-1}(F)\right) \neq \phi$. Hence, it follows that $x \in (i,j)-S_C Cl\left(X-f^{-1}(F)\right)$, but $x \in f^{-1}(F)$ and hence $x \in (i,j)-S_C Cl\left(f^{-1}(F)\right)$. This means that $x \in (i,j)-S_C Bd\left(f^{-1}(F)\right)$.

($\leftarrow$): *Suppose that* $x \in (i,j)-S_CBd(f^{-1}(F))$ *for some* $j-$ *regular closed subsets* F *of* Y *containing* $f(x)$. *If* f *is* $(i,j)-\theta S_C-$ *continuous at* x. *Then by Proposition 4.8, there exists an* $(i,j)-S_C-$ *open set* U *of* X *containing* x *such that* $f(U) \subseteq F$. *Then* $U \subseteq f^{-1}(F)$. *This shows that* $x \in (i,j)-S_CInt(f^{-1}(F))$. *Therefore,* $x \notin (i,j)-S_CBd(f^{-1}(F))$ *which is contradiction. Hence,* f *is not* $(i,j)-\theta S_C-$ *continuous.*

Corollary 4.20: If (X,τ_i) *is a* T_1- *space , then the following properties are equivalent for a function* $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$.

f *is* $(i,j)-\theta S_C-$ *continuous (resp.,* $(i,j)-S_C-$ *continuous).*

f *is* $j-$ *weakly* $\theta-$ *irresolute (resp.,* $j-$ *semi – continuous).*

Proof: Follows from Theorem 2.9.

Proposition 4.21: Let (X,σ_j) *be an extermally disconnected space,* (X,τ_i) *be a* T_1- *space, and* $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_{1s},\sigma_{2s})$ *is* $j-$ *semi – continuous, then* f *is* $(i,j)-\theta S_C-$ *continuous.*

Proof: Let F *be any* $j-$ *regular closed set of* Y. *Since* (Y,σ_j) *is extermally disconnected, so* $F \in j-RO(Y)$ *and by hypothesis,* $f^{-1}(F)$ *is* $j-$ *semi – open set of* X. *Since* (X,τ_i) *is* T_1- *space, then* $f^{-1}(F)$ *is* $(i,j)-S_C-$ *open, by Proposition 4.8 part (5),* f *is* $(i,j)-\theta S_C-$ *continuous.*

Proposition 4.22: Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ *be an* $(i,j)-\theta S_C-$ *continuous function and* Y *is* $j-$ *preopen subset of a bitopological space* Z, *then* $f:X\rightarrow Z$ *is* $(i,j)-\theta S_C-$ *continuous.*

Proof: Let V *be any* $j-$ *regular closed subset of* Z. *Since* Y *is* $j-$ *preopen set, then by Theorem 2.11* $V\cap Y$ *is* $j-$ *regular closed set in* Y. *Since,* $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ *is* $(i,j)-\theta S_C-$ *continuous, then by Proposition 4.8,* $f^{-1}(V\cap Y)$ *is an* $(i,j)-S_C-$ *open set in* X. *But* $f(x)\in Y$ *for each* $x\in X$, *thus* $f^{-1}(V)=f^{-1}(V\cap Y)$ *is an* $(i,j)-S_C-$ *open set in* X. *Therefore, by Proposition 4.8,* $f:X\rightarrow Z$ *is* $(i,j)-\theta S_C-$ *continuous.*

Proposition 4.23: A function $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous), if for each* $x\in X$, *there exists an* $i-$ *regular closed and* $j-$ *regular closed set* A *of* X *containing* x *such that* $f|A:A\rightarrow Y$ *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous).*

Proof: Let $x\in X$, *then by hypothesis there exists a subset* A *of* X *which is both* $i-$ *regular closed and* $j-$ *regular closed set containing* x *such that* $f|A:A\rightarrow Y$ *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous).*

Let V *be any* $j-$ *open (resp.,* $j-$ *semi – open) set of* Y *containing* $f(x)$, *there exists an* $(i,j)-S_C-$ *open set* U *in* A *containing* x *such that* $(f|A)(U)\subseteq V$ *(resp.,* $(f|A)(U)\subseteq j-Cl(V)$*). Since,* A *is* $i-$ *regular closed and* $j-$ *regular closed set, then by Theorem 2.11,* U *is* $(i,j)-S_C-$ *open set in* X. *Hence,* $f(U)\subseteq V$ *(resp.,* $f(U)\subseteq j-Cl(V)$*). This shows that* f *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous).*

Proposition 4.24: If $X=A\cup B$, *where* A *and* B *are both* $i-$ *regular closed and* $j-$ *regular closed sets and* $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ *is a function such that both* $f|A$ *and* $f|B$ *are* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous), then* f *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous).*

Proof: Let V *be any* $j-$ *open (resp.,* $j-$ *regular closed) set of* Y. *Then* $f^{-1}(V)=(f|A)^{-1}(V)\cup(f|B)^{-1}(V)$, *since* $f|A$ *and* $f|B$ *are* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous), then by Proposition 3.2 (resp., Proposition 4.8),* $(f|A)^{-1}(V)$ *and* $(f|B)^{-1}(V)$ *are* $(i,j)-S_C-$ *open sets in* A *and* B *respectively. Since* $A,B\in i-RC(X)\cap j-RC(X)$, *then by Theorem 2.11,* $(f|A)^{-1}(V)$ *and* $(f|B)^{-1}(V)$ *are* $(i,j)-S_C-$ *open sets in* X. *Since the union of two* $(i,j)-S_C-$ *open sets is* $(i,j)-S_C-$ *open. Hence* $f^{-1}(V)$ *is* $(i,j)-S_C-$ *open set in* X. *Therefore by Proposition 3.2 (resp., Proposition 4.8),* f *is* $(i,j)-S_C-$ *continuous (resp.,* $(i,j)-\theta S_C-$ *continuous).*

Proposition 4.25: Let $f:(X,\tau_1,\tau_2)\rightarrow(Y,\sigma_1,\sigma_2)$ *and* $g:(Y,\sigma_1,\sigma_2)\rightarrow(Z,\eta_1,\eta_2)$ *be functions. Then the composition function* $gof:X\rightarrow Z$ *is* $(i,j)-S_C-$ *continuous, if* f *and* g *satisfies one of the following conditions:*

f *is* $(i,j)-S_C-$ *continuous and* g *is* $j-$ *continuous.*

f *is* $(i,j)-\theta S_C-$ *continuous and* g *is* $j-\theta s-$ *continuous.*

f *is* $(i,j)-\theta S_C-$ *continuous and* g *is* $j-RC-$ *continuous.*

Proof: 1) Let W be any $j-$ open subset of Z. Since g is $j-$ continuous, so $g^{-1}(W)$ is $j-$ open subset of Y. Since, f is $(i,j)-S_C-$ continuous, then by Proposition 3.2, $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, by Proposition 3.2, gof is $(i,j)-S_C-$ continuous.

2) Let W be any $j-$ open subset of Z. Since, g is $j-\theta s-$ continuous, so $g^{-1}(W)$ is $j-\theta-$ semi – open subset of Y. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.8 part (6), $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, by Proposition 4.8, gof is $(i,j)-S_C-$ continuous.

3) Let W be any $j-$ open subset of Z. Since, g is $j-RC-$ continuous, so $g^{-1}(W)$ is $j-$ regular closed subset of Y. Since, f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.8 part (5), $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, by Proposition 4.8, gof is $(i,j)-S_C-$ continuous.

Proposition 4.26: Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2)\to(Z,\eta_1,\eta_2)$ be two functions. Then the composition function $gof:X\to Z$ is $(i,j)-\theta S_C-$ continuous, if f and g satisfies one of the following conditions:

f is $(i,j)-S_C-$ continuous and g is $j-(\theta,s)-$ continuous.

f is $(i,j)-\theta S_C-$ continuous and g is $j-\theta-$ irresolute.

f is $(i,j)-\theta S_C-$ continuous and g is $j-R-$ Map.

Proof:

Let W be any $j-$ regular closed subset of Z. Since g is $j-(\theta,s)-$ continuous, then $g^{-1}(W)$ is $j-$ open subset of Y. Since f is $(i,j)-S_C-$ continuous, then by Proposition 3.2, $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, by Proposition 4.8, gof is $(i,j)-\theta S_C-$ continuous.

Let W be any $j-$ regular closed subset of Z. Since g is $j-\theta-$ irresolute, then $g^{-1}(W)$ is $j-\theta-$ semi – open subset of Y. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.8 part (6), $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, gof is $(i,j)-\theta S_C-$ continuous.

Let W be any $j-$ regular closed subset in Z. Since g is $j-R-$ Map, then $g^{-1}(W)$ is $j-$ regular closed in Y. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.8, $(gof)^{-1}(W)=f^{-1}(g^{-1}(W))$ is $(i,j)-S_C-$ open subset in X. Therefore, gof is $(i,j)-\theta S_C-$ continuous.

Proposition 4.27: If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous and (Y,σ_j) is strongly $s-$ regular space, then f is $(i,j)-S_C-$ continuous.

Proof: Let $x\in X$ and let V be any $j-$ open set in Y containing $f(x)$. The strongly $s-$ regular of Y gives that, there exists a $j-$ regular closed subset F of Y such that $f(x)\in F\subseteq Y$. Since f is $(i,j)-\theta S_C-$ continuous, then by Proposition 4.2, there exists an $(i,j)-S_C-$ open set U of X containing x such that $f(U)\subseteq F\subseteq V$. Therefore, f is $(i,j)-S_C-$ continuous.

Proposition 4.28: If $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is $(i,j)-\theta S_C-$ continuous injection and (Y,σ_j) is $s-$Urysohn space, then (X,τ_j) is semi– Hausdorff and (X,τ_i) is T1 space.

Proof: Since f is injective, it follows that $f(x_1)\neq f(x_2)$, for any two distinct point x_1 and x_2. Since, (Y,σ_j) is $s-$ Urysohn space, then there exist $j-$ semi– open sets V_1 and V_2 of Y such that $f(x_1)\in V_1$, $f(x_2)\in V_2$ and $j-Cl(V_1)\cap j-Cl(V_2)=\phi$. Since f is $(i,j)-\theta S_C-$ continuous, then there exist an $(i,j)-S_C-$ open sets U_1 and U_2 of X containing x_1 and x_2 respectively, such that $f(U_1)\subseteq j-Cl(V_1)$ and $f(U_2)\subseteq j-Cl(V_2)$.

Hence, $U_1\cap U_2=\phi$. Since U_1 and U_2 are $(i,j)-S_C-$ open sets, then U_1 and U_2 are $j-$ semi – open sets. Therefore, (X,τ_j) is semi – Hausdorff . Also there exist two disjoint $i-$closed sets F_1 and F_2 containing x_1 and x_2 . Hence (X,τ_i) is T1 space.

Proposition 4.29: If $f_i:X_i\to Y_i$ is $(i,j)-S_C-$ continuous (resp., $(i,j)-\theta S_C-$ continuous) functions, for $i=1,2$. Let $f:X_1\times X_2\to Y_1\times Y_2$ be a function defined as follows: $f(x_1,x_2)=(f_1(x_1),f_2(x_2))$. Then f is $(i,j)-S_C-$ continuous (resp., $(i,j)-\theta S_C-$ continuous).

Proof: Let $R_1\times R_2\subseteq Y_1\times Y_2$, where R_i is $j-$ open (resp., $j-$regular closed) sets in Y_i, for $i=1,2$. Then $f^{-1}(R_1\times R_2)=f^{-1}(R_1)\times f^{-1}(R_2)$. Since f_i is $(i,j)-S_C-$continuous (resp., $(i,j)-\theta S_C-$ continuous), for $i=1,2$, then by Proposition 3.2 (resp., Proposition 4.8) and Proposition 2.13 $f^{-1}(R_1\times R_2)$ is an $(i,j)-S_C-$ open set in $X_1\times X_2$. Now, if R is *any* $j-$ *open (resp., j − regular closed) subset of* $Y_1 \times Y_2$, *then* $f^{-1}(R) = f^{-1}(\cup R_\alpha)$, *where* $\cup R_\alpha$ *is of the form* $R_{\alpha_1} \times R_{\alpha_2}$. *Therefore,* $f^{-1}(R) = \cup f^{-1}(R_\alpha)$ *is an* $(i,j)-S_C-$ *open set in* $X_1 \times X_2$. *Which completes the proof.*

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الدوال المستمرة -(i,j)-S_c والدوال المستمرة -(i,j)-θS_c

الملخص

في هذا البحث ادخلنا نمطا جديدا من الاستمرارية في الفضاءات الثنائية التوبولوجي سميت بالاستمرارية -(i,j)-S_c والاستمرارية -(i,j)-θS_c . ناقشنا العلاقات بين هذه الانماط وانماط اخرى معروفة من الدوال المستمرة.

نەخشێن بەردەوام -(i,j)-S_c نەخشێن بەردەوام -(i,j)-θS_c

كورتى

دڤێ ڤەكولينێدا مە دوو جورێن نوى ژ بەردەواميێ د ڤالاهيێن دوو توپولوجيدا دانە نياسين بناڤێ بەردەواميا -(i,j)-S_c و بەردەواميا -(i,j)-θS_c . مە پەيوەنديێن دناڤ بەرا ڤان جورێن بەردەواميا وجورێن دى ژ نەخشێن بەردەوام گفتوگو كرن.