On Pre-γ-I-Open Sets In Ideal Topological Spaces

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ABSTRACT

In this paper, we introduce and study the notion of pre-γ-I-open sets in ideal topological space.

Keywords:γ-open, pre-γ-I-open sets.

1.INTRODUCTION

In 1992, Jankovic and Hamlett introduced the notion of I-open sets in topological spaces via ideals. Dontchevin 1999 introduced pre-I-open sets, Kasaharain 1979 defined an operation α on a topological space to introduce α-closed graphs. Following the same technique, Ogata in 1991defined an operation γ on a topological space and introduced γ-open sets. In this paper, some relationships of pre-γ-I-open, pre-I-open, preopen, pre-γ-open, γ-p-open, γ-preopen, I-open, δ_I-open, R-I-open, α-I-open, semi-I-open, b-I-open and weakly I-local closed sets in ideal topological spaces are discussed.

2.PRELIMINARIES

Throughout this paper, (X, τ) and (Y, σ) stand for topological spaces with no separation axioms assumed unless otherwise stated. For a subset A of X, the closure of A and the interior of A will be denoted by $Cl(A)$ and $Int(A)$, respectively. Let (X, τ) be a topological space and Aa subset of X. A subset A of a space (X, τ) is said to be regular open [N. V.Velicko, 1968] if $A = Int(Cl(A))$. A is called δ-open [N. V.Velicko, 1968] if for each $x \in A$ there exists a regular open set G such that $x \in G \subseteq A$.An operation γ [S. Kasahara, 1979] on a topology τ is a mapping from τ in to power set $P(X)$ of X such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V. A subset $A$of X with an operation γ on τ is called γ-open [H. Ogata, 1991] if for each $x \in A$, there exists an openset U such that $x \in U$ and $\gamma(U) \subseteq A$. Then, τ_γ denotes the set of all γ-open set in X.Clearly $\tau_\gamma \subseteq \tau$. Complements of γ-open sets are called γ-closed. The τ_γ-interior [G. SaiSundara Krishnan, 2003] of A is denoted by τ_γ-$Int(A)$ and defined to be the union of all γ-open sets of X containedin A. The τ_γ-closure [H. Ogata, 1991] of A is denoted by τ_γ-$Cl(A)$ and defined to be the intersectionof all γ-closed sets containing A. A topological space (X, τ) with an operation γ on τ is said to be γ-regular [H. Ogata, 1991] if for each $x \in X$ and for each open neighborhood V of x,there exists an open neighborhood U of x such that $\gamma(U)$ contained in V. It is also tobe noted that $\tau = \tau_\gamma$if and only if X is a γ-regular space [H. Ogata, 1991].

An ideal is defined as a nonempty collection I of subsets X satisfying the followingtwo conditions:
1. If $A \in I$ and $B \subseteq A$, then $B \in I$.
2.If$A \in I$ and $B \in I$, then $A \cup B \in I$.

For an ideal I on (X, τ), (X, τ, I) is called an ideal topological space or simply an ideal space. Given a topological space (X, τ) with an ideal I on X and if $P(X)$ is the set of all subsets of X, a set operator $(.)^* : P(X) \rightarrow P(X)$ called a local function [E. Hayashi, 1964], [K. Kuratowski, 1966] of A with respect to τ and I is defined as follows for a subset A of X, $A^*(I, \tau) = \{x \in X : U \cap A \notin I$ for each neighborhood U of $x\}$. A Kuratowski closure operator $Cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the $*$-topology, finer than τ, is defined by $Cl^*(A) = A \cup A^*(I, \tau)$ [D. Jankovic and T. R. Hamlett, 1990]. We will simply write $A^*$for $A^*(I, \tau)$ and $\tau^*$for $\tau^*(I, \tau)$.
Recall that $A \subseteq (X, \tau, I)$ is called $*$-dense-in-itself [E. Hayashi, 1964] (resp. τ^*-closed [D. Jankovic and T. R. Hamlett, 1990] and $*$-perfect [E. Hayashi, 1964]) if $A \subseteq A^*$ (resp. $A^* \subseteq A$ and$A = A^*$).

Definition 2.1.A subset A of an ideal topological space (X, τ, I) is said to be
1. preopen [A. S. Mashhour, M. E. Abd El-Monsef and S. N. El-Deeb, 1982] if $A \subseteq Int(Cl(A))$.
2. pre-γ-open [H. Z. Ibrahim, 2012] if $A \subseteq \tau_\gamma$-$Int(Cl(A))$.
3. γ-preopen [G. S. S. Krishnan and K. Balachandran, 2006] if $A \subseteq \tau_\gamma$-$Int(\tau_\gamma$-$Cl(A))$.
4. γ-p-open [A. B. Khalaf and H. Z. Ibrahim, 2011] if $A \subseteq Int(\tau_\gamma$-$Cl(A))$.
5. I-open [D. Jankovic and T. R. Hamlett, 1992] if $A \subseteq Int(A^*)$.
6. R-I-open [S. Yuksel, A. Acikgoz and T. Noiri, 2005] if $A = Int(Cl^*(A))$.

7. pre-I-open [J. Dontchev, 1999] if $A \subseteq Int(Cl^*(A))$.
8. semi-I-open [E. Hatir and T. Noiri, 2002] if $A \subseteq Cl^*(Int(A))$.
9. α-I-open [E. Hatir and T. Noiri, 2002] if $A \subseteq Int(Cl^*(In(A)))$.
10. b-I-open [A. C. Guler and G. Aslim, 2005] if $A \subseteq Int(Cl^*(A)) \cup Cl^*(Int(A))$.
11. Weakly$I$-local closed [A. Keskin, T. Noiri and S. Yuksel, 2004] if $A= U \cap K$, where U is an open set and K is a $*$-closedset in X.
12. Locally closed [N. Bourbaki, 1966] if $A= U \cap K$, where U is an open set and K is a closed set in$X$.

Definition 2.2.[S. Yuksel, A. Acikgoz and T. Noiri, 2005] A point x in an ideal space (X, τ, I) is called a δ_I-cluster point of A if $Int(Cl^*(U)) \cap A \neq \phi$ for each neighborhood U of x. The set of all δ_I-cluster points of A is called the δ_I-closure of A and will be denoted by $\delta Cl_I(A)$. A is said to be δ_I-closedif $\delta Cl_I(A) = A$. The complement of a δ_I-closed set is called a δ_I-open set.

Lemma 2.3.[E. G. Yang, 2008] A subset V of an ideal space (X, τ, I) is a weakly I-local closed set if and only if there exists $K \in \tau$ such that $V = K \cap Cl^*(V)$.

Definition 2.4.[E. Ekici and T. Noiri, 2009] An ideal topological space (X, τ, I) is said to be $*$-extremally disconnected if the $*$-closure of every open subset V of X is open.

Theorem 2.5.[E. Ekici and T. Noiri, 2009] For an ideal topological space (X, τ, I), the following properties are equivalent:
1. X is $*$-extremally disconnected.
2. $Cl^*(Int(V)) \subseteq Int(Cl^*(V))$ for every subset V of X.

Lemma 2.6.[D. Jankovic and T. R. Hamlett, 1990] Let (X, τ, I) be an ideal topological space and A, B subsets of X. Then
1. If $A \subseteq B$, then $A^* \subseteq B^*$.
2. If $U \in \tau$, then $U \cap A^* \subseteq (U \cap A)^*$.
3. $A^*$is closed in (X, τ).

Recall that (X, τ) is called submaximal if every dense subset of X is open.

Lemma 2.7.[R. A. Mahmound and D. A. Rose, 1993] If (X, τ) is submaximal, then $PO(X, \tau) = \tau$.

Corollary 2.8.[J. Dontchev, 1999] If (X, τ) is submaximal, then for any ideal I on X, $PIO(X) = \tau$.
Where $PIO(X)$ is the family of all pre-I-open subsets of (X, τ, I).

Proposition 2.9.[H. Ogata, 1991] Lel$\gamma : \tau \rightarrow p(X)$ be a regular operation on τ. If A and B are γ-open, then $A \cap B$ is γ-open.

3.Pre-γ-I-Open Sets

Definition 3.1.A subset A of an ideal topological space (X, τ, I) with an operation γ on τ is called pre-γ-I-open if $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A))$.

We denote by $P\gamma IO(X, \tau, I)$ the family of all pre-γ-I-open subsets of (X, τ, I) or simply write $P\gamma IO(X, \tau)$ or $P\gamma IO(X)$ when there is no chance for confusion with the ideal.

Theorem 3.2. Every γ-open set is pre-γ-I-open.

Proof.Let (X, τ, I) be an ideal topological space and Aaγ-open set of X. Then $A = \tau_\gamma\text{-}Int(A) \subseteq \tau_\gamma\text{-}Int(A \cup A^*) = \tau_\gamma\text{-}Int(Cl^*(A))$.
The converse of the above theorem is not true in general as shown in the followingexample.

Example 3.3. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a, c\}\}$ and $I = \{\phi, \{b\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then $A = \{a, b\}$ is a pre-γ-I-open set which is not γ-open.

Theorem 3.4. Every pre-γ-I-open set is pre-γ-open.

Proof.Let (X, τ, I) be an ideal topological space and Aa pre-γ-I-open set of X. Then,
$A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) \subseteq \tau_\gamma\text{-}Int(Cl(A))$.
The converse of the above theorem is not true in general as shown in the followingexample.

Example 3.5. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{b, c\}\}$ and $I = \{\phi, \{c\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Set $A = \{c\}$, since $A^* = \phi$and$Cl^*(A) = A$, then Ais a pre-γ-open set which is not pre-γ-I-open.

Theorem 3.6. Every pre-γ-I-open set is pre-I-open.

Proof.Let (X, τ, I) be an ideal topological space and Aa pre-γ-I-open set of X. Then,
$A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) \subseteq Int(Cl^*(A))$.
The converse of the above theorem is not true in general as shown in the followingexample.

Example 3.7. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{c\}\}$ and $I = \{\phi, \{c\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then $A = \{c\}$ is a pre-I-open set which is not pre-γ-I-open.

Theorem 3.8. Every pre-γ-I-open set is γ-preopen.

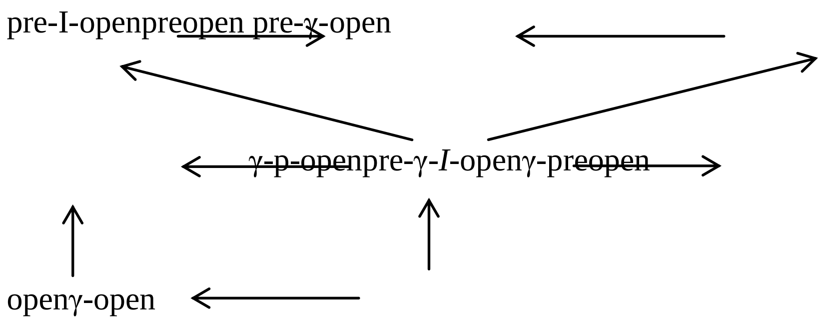
Proof.Let (X, τ, I) be an ideal topological space and Aa pre-γ-I-open set of X. Then,
$A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) \subseteq \tau_\gamma\text{-}Int(Cl(A)) \subseteq \tau_\gamma\text{-}Int(\tau_\gamma\text{-}Cl(A))$.
The converse of the above theorem is not true in general as shown in the followingexample.

Example 3.9. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{b\}, \{a, b\}\}$ and $I = \{\phi, \{b\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then A

$= \{b, c\}$ is a γ-preopen set which is not pre-γ-I-open.

Theorem 3.10. Every pre-γ-I-open set is γ-p-open.**Proof**.Let (X, τ, I) be an ideal topological space and Aa pre-γ-I-open set of X. Then,$A \subseteq \tau_\gamma$-$Int(Cl^*(A)) \subseteq \tau_\gamma$-$Int(Cl(A)) \subseteq Int(\tau_\gamma$-$Cl(A))$.

The converse of the above theorem is not true in general as shown in the followingexample.**Example 3.11.**Consider $X = \{a, b, c, d\}$ with $\tau = P(X)$ and $I = \{\phi\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Then $A = \{c, d\}$ is a γ-p-open set which is not pre-γ-I-open.**Remark 3.12.**We have the following implications but none of this implications are reversible.



The intersection of two pre-γ-I-open sets need not be pre-γ-I-open as shown in the following example.

Example 3.13. Consider $X = \{a, b, c\}$ with $\tau = \{\phi, X, \{a, c\}\}$ and $I = \{\phi, \{b\}\}$. Define an operation γ on τ by $\gamma(A) = X$ for all $A \in \tau$. Set $A = \{a, b\}$ and $B = \{b, c\}$. Since $A^* = B^* = X$, then both A and B are pre-γ-I-open. But on the other hand $A \cap B = \{b\} \notin P\gamma IO(X, \tau)$.

Theorem 3.14. Let (X, τ, I) be an ideal topological space and $\{A_\alpha: \alpha \in \Delta\}$ a family of subsets of X, where Δ is an arbitrary index set. Then,

1. If $A_\alpha \in P\gamma IO(X, \tau)$ for all $\alpha \in \Delta$, then $\cup_{\alpha\in\Delta} A_\alpha \in P\gamma IO(X, \tau)$.
2. If $A \in P\gamma IO(X, \tau)$ and $U \in \tau_\gamma$, then $A \cap U \in P\gamma IO(X, \tau)$. Where γ is regular operation on τ.

Proof.

1. Since $\{A_\alpha: \alpha \in \Delta\} \subseteq P\gamma IO(X, \tau)$, then $A_\alpha \subseteq \tau_\gamma$-$Int(Cl^*(A_\alpha))$ for each $\alpha \in \Delta$. Then we have $\cup_{\alpha\in\Delta} A_\alpha \subseteq \cup_{\alpha\in\Delta}\tau_\gamma$-$Int(Cl^*(A_\alpha)) \subseteq \tau_\gamma$-$Int(\cup_{\alpha\in\Delta} Cl^*(A_\alpha)) \subseteq \tau_\gamma$-$Int(Cl^*(\cup_{\alpha\in\Delta} A_\alpha))$.
This shows that $\cup_{\alpha\in\Delta} A_\alpha \in P\gamma IO(X, \tau)$.
2. By the assumption, $A \subseteq \tau_\gamma$-$Int(Cl^*(A))$ and $U = \tau_\gamma$-$Int(U)$. Thus using Lemma 2.6, we have $A \cap U \subseteq \tau_\gamma$-$Int(Cl^*(A)) \cap \tau_\gamma$-$Int(U) = \tau_\gamma$-$Int(Cl^*(A) \cap U) = \tau_\gamma$-$Int((A^* \cup A) \cap U) = \tau_\gamma$-$Int((A^* \cap U) \cup (A \cap U)) \subseteq \tau_\gamma$-$Int((A \cap U)^* \cup (A \cap U)) = \tau_\gamma$-$Int(Cl^*(A \cap U))$.
This shows that $A \cap U \in P\gamma IO(X, \tau)$.

Proposition 3.15. For an ideal topological space (X, τ, I) with an operation γ on τ and $A \subseteq X$ we have:1. If $I = \{\phi\}$, then A is pre-γ-I-open if and only if A is pre-γ-open.2. If $I = P(X)$, then $P\gamma IO(X) = \tau_\gamma$.**Proof**.1. By Theorem 3.4, we need to show only sufficiency. Let $I = \{\phi\}$, then $A^* = Cl(A)$ for every subset A of X. Let A be pre-γ-open, then $A \subseteq \tau_\gamma$-$Int(Cl(A)) = \tau_\gamma$-$Int(A^*) \subseteq \tau_\gamma$-$Int(A \cup A^*) = \tau_\gamma$-$Int(Cl^*(A))$ and hence A is pre-γ-I-open.2. Let $I = P(X)$, then $A^* = \phi$ for every subset A of X. Let A be any pre-γ-I-open set, then $A \subseteq \tau_\gamma$-$Int(Cl^*(A)) = \tau_\gamma$-$Int(A \cup A^*) = \tau_\gamma$-$Int(A \cup \phi) = \tau_\gamma$-$Int(A)$ and hence A is γ-open. By Theorem 3.2, we obtain $P\gamma IO(X) = \tau_\gamma$.

Remark 3.16.

1. If a subset A of a γ-regular space (X, I, τ) is open then A is pre-γ-I-open.
2. If a subset A of a submaximal space (X, I, τ) is pre-γ-I-open then A is open.
3. If (X, I, τ) is γ-regular space and $I = P(X)$, then A is pre-γ-I-open if and only if A is open.

Remark 3.17. Let (X, I, τ) be a γ-regular space and $I = P(X)$. Then

1. If A is R-I-open then A is pre-γ-I-open.
2. If A is δ_I-open then A is pre-γ-I-open.
3. If A is regular open then A is pre-γ-I-open.
4. If A is δ-open then A is pre-γ-I-open.

Remark 3.18. For an ideal topological space (X, τ, I) with an operation γ on τ and $I = P(X)$ we have:

1. If A is pre-γ-I-open then A is open.
2. If A is pre-γ-I-open then A is α-I-open.
3. If A is pre-γ-I-open then A is semi-I-open.

Proposition 3.19. Let (X, τ, I) be an ideal topological space and Aa subset of X. If A is closed and pre-γ-I-open, then A is R-I-open.

Proof.Let A be pre-γ-I-open, then we have $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) \subseteq Int(Cl^*(A)) \subseteq Int(Cl(A)) \subseteq Cl(A) = A$ and hence A is R-I-open.

Remark 3.20. Let (X, I, τ) be γ-regular space. If $A \subseteq (X, I, \tau)$ is R-I-open, then A is pre-γ-I-open.

Remark 3.21. If (X, I, τ) is γ-regular space and $I = \{\phi\}$. Then

1. A is pre-γ-I-open if and only if A is preopen.
2. A is pre-γ-I-open if and only if A is γ-preopen.
3. A is pre-γ-I-open if and only if A is γ-p-open.

Proposition 3.22. Let (X, τ, I) be an ideal topological space and Aa subset of X. If $I = \{\phi\}$ and A is pre-γ-I-open, then A is I-open.

Proof.Let A be pre-γ-I-open, then we have $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) \subseteq \tau_\gamma\text{-}Int(Cl(A)) \subseteq \tau_\gamma\text{-}Int(A^*) \subseteq Int(A^*)$ and hence A is I-open.

Remark 3.23. If (X, I, τ) is a γ-regular space and A is δ_I-open then A is pre-γ-I-open.

Remark 3.24. If (X, I, τ) is γ-regular then A is pre-γ-I-open if and only if A is pre-I-open.

Proposition 3.25. If $A \subseteq (X, I, \tau)$ is $*$-perfect and pre-γ-I-open, then A is γ-open.

Proof.Let A be $*$-perfect, then $A = A^*$and $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) = \tau_\gamma\text{-}Int(A \cup A^*) = \tau_\gamma\text{-}Int(A \cup A) = \tau_\gamma\text{-}Int(A)$ and hence A is γ-open.

Remark 3.26. If $A \subseteq (X, I, \tau)$ is $*$-perfect and pre-γ-I-open, then A is open.

Proposition 3.27. If A is τ^*-closed in (X, I, τ) and pre-γ-I-open, then A is γ-open.

Proof.Let A be pre-γ-I-open, then $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) = \tau_\gamma\text{-}Int(A \cup A^*) = \tau_\gamma\text{-}Int(A)$ and hence A is γ-open.

Remark 3.28. If A is τ^*-closed in (X, I, τ) and pre-γ-I-open, then A is open.

Proposition 3.29. If A is $*$-perfect in (X, I, τ) and pre-γ-I-open, then A is I-open.

Proof.Let A be pre-γ-I-open, then $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) = \tau_\gamma\text{-}Int(A \cup A^*) = \tau_\gamma\text{-}Int(A^*) \subseteq Int(A^*)$ and hence A is I-open.

Proposition 3.30. If A is $*$-dense-in-itself in (X, I, τ) and pre-γ-I-open, then A is I-open.

Proof.Let A be pre-γ-I-open, then $A \subseteq \tau_\gamma\text{-}Int(Cl^*(A)) = \tau_\gamma\text{-}Int(A \cup A^*) = \tau_\gamma\text{-}Int(A^*) \subseteq Int(A^*)$ and hence A is I-open.

Proposition 3.31. If a subset A of a $*$-extremally disconnected γ-regular space (X, I, τ) is α-I-open then A is pre-γ-I-open.

Proof.Let A be α-I-open, then $A \subseteq Int(Cl^*(Int(A))) \subseteq Cl^*(Int(A)) \subseteq Int(Cl^*(A)) = \tau_\gamma\text{-}Int(Cl^*(A))$ and hence A is pre-γ-I-open.

Proposition 3.32. If a subset A of a $*$-extremally disconnected γ-regular space $(X, I, \tau)$is semi-I-open then A is pre-γ-I-open.

Proof.Let A be semi-I-open, then $A \subseteq Cl^*(Int(A)) \subseteq Int(Cl^*(A)) = \tau_\gamma\text{-}Int(Cl^*(A))$ and hence A is pre-γ-I-open.

Proposition 3.33. If a subset A of a $*$-extremally disconnected γ-regular space $(X, I, \tau)$is b-I-open and $I = P(X)$, then A is pre-γ-I-open.

Proof.Let A be b-I-open, then $A \subseteq Int(Cl^*(A)) \cup Cl^*(Int(A)) \subseteq Int(A \cup A^*) \cup Cl^*(Int(A)) \subseteq Int(A \cup \phi) \cup Cl^*(Int(A)) \subseteq Int(A) \cup Cl^*(Int(A)) \subseteq Int(A) \cup (Int(A) \cup Int(A)^*) \subseteq Int(A) \cup Int(A)^* \subseteq Cl^*(Int(A)) \subseteq Int(Cl^*(A)) = \tau_\gamma\text{-}Int(Cl^*(A))$ and hence A is pre-γ-I-open.

Theorem 3.34. Let (X, I, τ) be a $*$-extremally disconnected γ-regular ideal space and $V \subseteq X$, the following properties are equivalent:

1. V is a γ-open set.
2. V is α-I-open and weakly I-local closed.
3. V is pre-γ-I-open and weakly I-local closed.
4. V is pre-I-open and weakly I-local closed.
5. V is semi-I-open and weakly I-local closed.
6. V is b-I-open and weakly I-local closed.

Proof.(1) $\Rightarrow$(2): It follows from the fact that every γ-open set is open and every openset is α-I-open and weakly I-local closed.

(2) $\Rightarrow$(3): It follows from Proposition 3.31.

(3) $\Rightarrow$(4), (4) $\Rightarrow$(5) and (5) $\Rightarrow$(6): Obvious.

(6) $\Rightarrow$(1): Suppose that V is a b-I-open set and a weakly I-local closed set in X. It follows that $V \subseteq Cl^*(Int(V)) \cup Int(Cl^*(V))$. Since V is a weakly I-local closed set, then there exists an open set G such that $V = G \cap Cl^*(V)$. It follows from Theorem 2.5 that $V \subseteq G \cap (Cl^*(Int(V)) \cup Int(Cl^*(V)))$

$= (G \cap Cl^*(Int(V))) \cup (G \cap Int(Cl^*(V)))$

$\subseteq (G \cap Int(Cl^*(V)) \cup (G \cap Int(Cl^*(V)))$

$= Int(G \cap Cl^*(V)) \cup Int(G \cap Cl^*(V))$

$= Int(V) \cup Int(V)$

$= Int(V)$

$= \tau_\gamma\text{-}Int(V)$.

Thus, $V \subseteq \tau_\gamma\text{-}Int(V)$ and hence V is a γ-open set in X.**Theorem 3.35.**Let (X, I, τ) be a $*$-extremally disconnected γ-regular ideal space and $V \subseteq X$, the following properties are equivalent:1. V is a γ-open set.2. V is α-I-open and a locally closed set.3. V is pre-γ-I-open and a locally closed set.4. V is pre-I-open and a locally closed set.5. V is semi-I-open and a locally closed set.6. V is b-I-open and a locally closed set.

Proof.By Theorem 3.34, it follows from the fact that every open set is locally closed and every locally closed set is weakly I-local closed.

Definition 3.36.A subset F of a space (X, τ, I) is said to be pre-γ-I-closed if its complement is pre-γ-I-open.

Theorem 3.37.A subset A of a space (X, τ, I) is pre-γ-I-closed if and only if $\tau_\gamma\text{-}Cl(Int^*(A)) \subseteq A$.

Proof.Let A be a pre-γ-I-closed set of (X, τ, I). Then $X-A$ is pre-γ-I-open and hence $X-A \subseteq \tau_\gamma\text{-}Int(Cl^*(X-A)) = X-\tau_\gamma\text{-}Cl(Int^*(A))$. Therefore, we have $\tau_\gamma\text{-}Cl(Int^*(A)) \subseteq A$.

Conversely, let $\tau_\gamma\text{-}Cl(Int^*(A)) \subseteq A$. Then $X-A \subseteq \tau_\gamma\text{-}Int(Cl^*(X-A))$ and hence $X-A$ is pre-γ-I-open. Therefore, A is pre-γ-I-closed.

Theorem 3.38. If a subset A of a space (X, τ, I) is pre-γ-I-closed, then $Cl(\tau_\gamma\text{-}Int(A)) \subseteq A$.

Proof.Let A be any pre-γ-I-closed set of (X, τ, I). Since $\tau^*(I)$ is finer than τ and τ is finer than τ_γ, we have $Cl(\tau_\gamma\text{-}Int(A)) \subseteq \tau_\gamma\text{-}Cl(\tau_\gamma\text{-}Int(A)) \subseteq \tau_\gamma\text{-}Cl(Int(A)) \subseteq \tau_\gamma\text{-}Cl(Int^*(A))$. Therefore, by Theorem 3.37, we obtain $Cl(\tau_\gamma\text{-}Int(A)) \subseteq A$.

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لسەر کومێنpre-γ-Iڤەکری ل ڤالاهییێن نموونەیی توبولوجی دا

کورتی

ژ ڤێ ڤەکولینێ جورەکێ ژ کوما ئـەم بدەنـە نیاسـین وخوانـدن بنـاڤێ کـومێن ڤـەکری ژ جـورێ pre-γ-*I* ل ڤالاهییێننموونەیی توبولوجی دا.

لملـخص

الغرض من هذا العمل هو تقديم و دراسة صنفمن المجموعات والتي اسميناها بالمجموعـات المفتوحـة من النمط **pre-γ-*I***في الفضاء التوبولوجي المثالي .