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journals.uoz.edu.krd

Science Journal of University of Zakho

Vol. 12, No.3, pp.345–355 July-September, 2024

p-ISSN: 2663-628X
e-ISSN: 2663-6298

VAR TIME SERIES ANALYSIS USING WAVELET SHRINKAGE WITH APPLICATION

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Received: 19 Apr., 2024 / Accepted: 2 July., 2024 / Published: 3 Aug., 2024. https://doi.org/10.25271/sjuoz.2024.12.3.1304

ABSTRACT:

This study investigates the VAR time series data of the overall expenditures and income in the Kurdistan Region of Iraq. It applies multivariate wavelet shrinkage within the VAR model, comparing it to traditional methods to identify the most appropriate model. The chosen model will then be used to predict general expenditures and revenues for the years 2022-2026. The analysis involved assessing the stationarity of the expenditure and revenue time series, which are interrelated variables during the interval 1997-2021, and identifying the overall trend through differencing to achieve stationarity. The proposed method incorporated multivariate wavelet shrinkage in the VAR model to address data contamination in expenditures and revenue using various wavelets like Coiflets, Daubechies, Symlets, and Fejér–Korovkin at different orders. Threshold levels were estimated using the SURE method and soft thresholding rules to denoise the data for the following analysis within the VAR model. Model selection was based on Akaike and Bayes information criteria. The analysis, conducted using MATLAB, indicated the superiority of the proposed method over traditional methods, forecasting a continued rise in expenditures and revenues for the Iraqi Kurdistan region from 2022 to 2026. The findings suggest that advanced techniques can offer more accurate economic forecasts, benefiting regional planning and policy-making.

KEYWORDS: Time Series, VAR, Wavelet, Threshold, Soft Rule.

1. INTRODUCTION

A wide range of disciplines, including statistics, inventory management, and economics, regularly use time series forecasting. There are several forecasting models, ranging from simple moving averages and linear regression through autoregressive integrated moving averages (ARIMA) and more sophisticated neural networks. These models examine the past to offer projections about the future. Time series are typically thought of as stationary random series because they are not always predictable (Zivot & Wang, 2003). Three factors must be taken into account when modeling time series: a deterministic function, white noise, and colored noise. A better model can be obtained for a time series by reducing its noise. Mathematical transformations are used like the Fourier and wavelet transforms to do this. A vector autoregression (VAR) approach is among the most efficient, versatile, and straightforward techniques for the analysis of multivariate time series. It makes it reasonable to expand the univariate autoregressive model into a dynamical multivariate time series. It has been demonstrated how effective the VAR model is in predicting and describing the dynamic behavior of financial and economic time series. It often provides predictions that outperform those derived from univariate time series models and intricate simultaneous equations models.

Many studies have been done using the VAR model. The researcher (C.A.D. Garcia, 2021) in his paper evaluated the impact of profit and accumulation on Colombia's growth rate from 1967-2019 using a VAR model. Findings showed that both variables are statistically significant and positively affect growth, with direct impacts on profit and accumulation rates and an inverse relationship between these variables. Other researchers used a VAR model to examine the impact of oil price shocks on stock returns in Latin American markets. The research reveals that structural demand shocks during the COVID-19 era have high standard deviations, and the pass-through effects on stock returns vary over different time frames. The study suggested that oil price impacts stock market returns based on time-frequency, aiding policymakers in restoring investor confidence and implementing risk mitigation strategies (J.C.T. Gaytan et al, 2023). Furthermore, some researchers compared VAR and ARIMAX models for time series analysis and forecasting using data from the Iraqi general budget (2004-2020), concluding that the VAR model was more efficient. Forecasts for 2021-2024 indicated a continued increase in foreign reserves and government spending (E.A. Haydier et al., 2023).

Because forecasts from VAR models may be formed conditional on the likely future courses of certain model variables, they are extremely flexible. The VAR model is utilized not only for data description and forecasting but also for structural inference and policy research. The causal consequences of unexpected shocks or innovations to defined variables on the variables in the model are summarized as a result of particular assumptions about the causal structure of the data under consideration. Typically, forecast error variance decompositions and impulse response functions are used to summarize these causal effects.

Reducing noise in data from time series is one of the primary issues. A novel and effective method for financial time series analysis is wavelet analysis, which lowers data noise (de-noise). Wavelets allow for the multi-level breakdown of a signal or time series. As a result, this breakdown makes the underlying signal's structure and any previously hidden patterns, periodicities, leaps, or singularities visible. In this work, we suggest techniques for reducing the influence of noise or impurity on multivariate time series data. We did this by using multivariate wavelets like Daubechies and Coiflets in conjunction with thresholding techniques such as Sure-Threshold and the use of a soft rule.

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1.1 Theoretical Aspect
The theoretical aspect presented some basic concepts on the subject of research from the statistical side, as shown in the following paragraphs.

1.2 Time Series
Time series is a set of observations grouped by time. Every observation takes place at some time T, where T is related to the range of permitted times. It is important to note that T might be either a continuous time series or a discrete time series, depending on the type of data. Time series can be useful for predictions. It acknowledges the range and uses it to project what is ahead. Time series analysis has also been used to examine the link between the selected data point and changes in other variables over the same period. The foundation of time series is comprised of two primary elements: the autoregressive (AR) model which generates forecasts by combining the historical values of the objective linearly, and the moving-average (MA) model modeling univariate time series. The moving-average model determines the output variable based on the present value of a stochastic term and its historical values (Raza et al. 2018). There are four ways to go about time-series forecasting:

1- The (ARIMA) process combines the AR and MA models with differences. ARIMA models arrange stages according to a linear function. The observations and residuals are from prior phases. This method works effectively for univariate time series with trends and no seasonal components.

2- The seasonal ARIMA or SARIMA model relies on previous observations, seasonal variations, and errors to guide subsequent steps. The SARIMA algorithm is used to fit a univariate time series with seasonal or trend components.

3- Vector Autoregression (VAR) models future steps in a time series using AR methods. It applies AR generalizations to multivariate forms.

4- Simple Exponential Smoothing (SES) is a technique that leverages prior data to create an exponentially weighted linear function for the next time step. This approach works well on univariate time series without trends or seasonal components.

1.3 Time Series Data
There are two types of time series data deterministic and nondeterministic:

1- Deterministic time series use analytical expressions. It lacks random or probabilistic components. In deterministic time series, derivative values at each point indicate the past and future.

2- Non-deterministic time series have a random component, making analytical expressions inaccurate. If one of the two requirements is satisfied, the facts might be considered non-deterministic. These instances may include missing data or a random function that created the data.

1.4 Time Series Analysis Techniques Using VAR Model
A multivariate time series model called the vector autoregressive (VAR) model connects the current observations of a variable to its historical observations as well as historical data of other variables in the system. Because they permit feedback between the model's variables, VAR models differ from univariate autoregressive models. A VAR model, for instance, might be used to demonstrate how the real gross domestic product (GDP) and policy rate are both functions of real GDP (Ali et al. 2022).

A methodical but adaptable technique for modeling complex behavior in the real world improved forecasting efficiency. the capacity to capture the entangled dynamics of time series data. VAR modeling takes several steps, and a full VAR analysis includes Specifying and estimating a VAR model, using inferences to check and revise the model (as needed), forecasting, and structural analysis.

The reduced form, the recursive form, and the structural VAR model are the three main categories of VAR models. The reduced form of each variable in VAR models is viewed as a function of its past values and the past values of other variables in the model (Omer et al. 2020).

Despite being the most straightforward VAR model, reduced-form models have the following disadvantages: Different contemporaneous variables have no common relationships. The error terms in different equations will be interrelated. As a result, we are unable to predict the effects of individual shocks on the system. All the elements of the reduced form model are present in recursive VAR models, but they additionally permit some variables to be functions of concurrent variables. We can model structural shocks using the recursive model by enforcing these short-run relationships. Restrictions in structural VAR models enable us to find causal links beyond those revealed by reduced-form or recursive models. It is possible to simulate and predict the effects of certain shocks, such as policy decisions, using these causal links.

A VAR model is composed of an equation framework that depicts the connections between several variables. We frequently employ specialized terminology when discussing VAR models to indicate:

- The number of endogenous variables used.
- The number of included autoregressive terms.

Lag selection is a crucial component of the VAR model formulation. In practical applications, we often select a maximum number of delays, Pmax, and assess the performance of the model by taking p=0,1, ..., pmax into account. The model VAR(p) that minimizes a certain lag selection criterion is then the best one to use. VAR models are characterized by their order, which refers to the number of earlier periods the model will use. A lag is the value of a variable in a previous period. So, in general, a pth-order VAR refers to a VAR model that includes lags for the last p periods. A p^{th}-order VAR is denoted "VAR(p)" and sometimes called "a VAR with p lags". A p^{th}-order VAR model is written as.

$$Y_t = \delta_t + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \ldots + \emptyset_p Y_{t-p} + \epsilon_t \qquad (1)$$

Where: $\emptyset_1, \emptyset_2, \ldots, \emptyset_p$ represents the autoregressive parameters, δ_t is trend. For example, the VAR(2) model can be express as:

$$Y_t = \delta_t + \emptyset_1 Y_{t-1} + \emptyset_2 Y_{t-2} + \epsilon_t \qquad (2)$$

$$\begin{bmatrix} y_t \\ y_{t-1} \end{bmatrix} = \begin{bmatrix} \delta_t \\ 0 \end{bmatrix} + \begin{bmatrix} \emptyset_1 & \emptyset_2 \\ I & 0 \end{bmatrix} \begin{bmatrix} Y_{t-1} \\ Y_{t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \qquad (3)$$

The following are the most typical lag selection criteria:

-Akaike (AIC)

-Schwarz-Bayesian (BIC)

-Hannan-Quinn (HQ).

Lag selection is now almost entirely automated, and these techniques are typically included in software. When estimating with a VAR model, it's important to carefully assess the number of variables included. Including more variables:

-Increases the number of coefficients to be estimated for each equation and each number of lags.

-Introduce additional estimation error.

VAR models are relatively simple to estimate, despite their seeming complexity. Given a few assumptions, the equation can be approximated using ordinary least squares, they are the conditional mean of the error term is zero, the model's variables are stationary, large outliers are improbable and no multicollinearity is ideal.

2. WAVELETS
Indeed, a wavelet is a small wave. A small wave grows and disintegrates in a brief amount of time. In contrast, a "big wave" unequivocally contradicts this idea. An example of a massive wave, the sine function fluctuates both upwards as well as

downwards when plotted against u∈(-∞,+∞) (Percival &Waldden, 2000). The mathematician Alfred Haar adopted and proposed the wavelet analysis concept in 1909. For processing and analysis, the wavelet transform WT represents a mathematical function that transforms the original data into a different domain (particularly with the time domain). The function as a model is appropriate for stationary as well as non-stationary time series data (Al Wadi et al. 2010); and (Brifcani & Al-Bamerni 2010). We begin with the Fourier transform (FT), which divides signals into many sets of basic functions and may shift and reverse the domain of a given signal from the time to the frequency. This is a formal mathematical statement of the transformation of the function x(t) into X(f):

$$\mathrm{X(f)} = \int_{-\infty}^{+\infty} \mathrm{x(t)e^{-iw(t)}dt} \qquad (4)$$

In this case, $i = \sqrt{-1}$ and $e^{i\theta} = cos\theta + isin\theta$. When a signal is changing over time, the FT loses effectiveness because it just provides information on frequency content and does not keep track of time. Because of this, the extended form of the transformation has been identified and is known as Gabor's adaption. It is said as follows:

$$STFT_X^{(W)}(t', f) = \int_t^{\infty} [X(t), W^*(t - t')]. e^{-2\pi f t} dt \quad (5)$$

Here, STFT stands for the Short-Time Fourier Transform, *X(t)* stands for the signal to be transformed, t' stands for the shift factor, w(t) stands for the window function, and * is the complex conjugate. The size and shape of the window limit the accuracy of STFT, even though it can adapt to frequency and time information. For instance, using time intervals repeatedly might result in very small windows and good time resolution. Employing signals for low frequency will result in poor frequency resolution due to the very short period of each window (Fugal, 2009). The WT thus appears to be a viable solution to the STFT problem. When evaluating the signal, we can obtain a different scale using this method, which represents scaled-version and it can be expressed as:

$$\Psi_*^{\psi}(x, s) = \int x(t). \psi_{\tau,s}^*(t) d(t) \qquad (6)$$

Where:

$$\psi_{\tau,s}^* = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\gamma}{s}\right) \qquad (7)$$

The scale and translation variables are represented by s and y, respectively. By putting the equation (7) in equation (6), we can obtain the continuous wavelet transform CWT, which can be described as the following formula:

$$CWT_x^{\psi}(\gamma, s) = \frac{1}{\sqrt{s}} \int x(t). \psi\left(\frac{t-\gamma}{s}\right) d(t) \quad (8)$$

It is obvious from equation (5) that the function of the analysis stands in for the wavelet. The CWT compares the determined signal to the stretched and shifted wavelet versions (Shahla et al. 2023). Dilation, often known as scaling, is a compression function. In mathematics, the term "CWT" refers to a non-numerical apparatus or instrument that produces an overcomplete representation of a signal by continuously adjusting the translation and scale parameters of wavelets. It is crucial to note that the data must be discretized to compute the WT numerically. When discretizing the CTW into Discrete Wavelet Transform DWT, one can obtain sufficient data for decomposition and synthesis. It significantly reduces computing time, is simpler to use, and can analyze signals at multiple frequencies and resolutions. It can also split the signal into rough approximations and detailed information. When employing discrete values, a wavelet is translated and dilated in DWT. Most of the time, the dilation is represented by a factor of two (Hubbard, 1996). Many types of wavelets can be used in data analysis, but after applying many attempts at different wavelets on the study data, it became clear that the most appropriate and best wavelets are the ones that will be discussed and employed in the application part of this paper and these wavelets are:

2.1 Daubechies Wavelets

Ingrid Daubechies, a scientist considered the founding father of wavelet study, is the name given to these wavelets. She created the so-called natural orthonormal wavelets with the characteristic of compact support in 1988, opening the way for the application of DWT (Kareem et al. 2020). The filters in this family are referred to by the initials DN or dbL1, where D and db refer to the researcher's last name Daubechies. For example, the second-order candidates in this family, represented by D4 and db2, are identical (Misiti et al. 1996). A Daubechies wavelet L1 contains N/2 vanishing moments, which is equivalent to half the number of filter points. Because db1 and the Haar wavelet are comparable, dbN is commonly used to represent the N family of wavelets, which includes the Haar wavelet.

It is commonly recognized that the wavelet functional—also referred to as the mother wavelet functional—and the scaling function is crucial to wavelet analysis. Equations (9) and (10) show that a set of *N* integer coefficients regulate each member wavelet, and for k = (0,1,..., N-1), the filter coefficients a_k , and a_{1k} coefficients are expressed as follows:

$$\Psi(u) = \sum_{k=2-N}^{1} (-1)^k \, a_{1-k} \emptyset(2u - k) \qquad ; (1 - \frac{N}{2}, \frac{N}{2}) \qquad (9)$$

$$\phi(u) = \sum_{k=0}^{N-1} (-1)^k \, a_k \emptyset(2u - k) \qquad ; (0, N-1) \qquad (10)$$

Equations (6) and (7) provide the wavelet function and scaling function of the Dubechies wavelet, with *u* representing the continuous time variable (Mustafa & Ali, 2013). Wavelets exhibit orthogonality, biorthogonality, compact support, non-symmetrical behavior, and other properties (Daubechies, 1992).

2.2 Coiflets Wavelets

In response to Coifman's 1989 suggestion to combine the vanishing moments of low-passing and high-passing filters (ϕ and Ψ) rather than concentrating just on Ψ, Daubechies created these wavelets. These wavelets are called Coif N, where *N* is the candidate order and Coif is the acronym for Coifman. The order of the candidates and the filter length are related. Additionally, the scaling functional ϕ has (L1 = 2 N-1) moments of vanishing, whereas the wavelet function Ψ has (L = 2N) moments of vanishing. Coiflet wavelets exhibit traits such as orthogonality, biorthogonality, compact support, and near-symmetry. For further information, please see (Daubechies, 1994).

2.3 Symlet Wavelets

As the name suggests, Symlet Wavelets are more symmetrical than Daubechies wavelets. The half-band filters can be factored in a variety of ways to create a decomposition filter and reconstruction filter that, when convolved, produce the same half-band filter. We can develop more symmetrical alternatives to the Daubechies filters for the longer half-band filters. Symlets wavelets have some properties such as orthogonality, biorthogonality, compact support, near from symmetry, …, etc. We refer the reader to (Daubechies, 1994) for more details.

2.4 Fejér–Korovkin Wavelet

Filter creation is critical to the success of any wavelet. The issue with traditional filters is that wavelets lose high-frequency resolution. To address this issue, "fkN" is an orthogonal wavelet transform based on Fejér-Korovkin filters. It achieves optimum frequency resolution using convolution kernels. Filters with N coefficients (e.g. 4, 6, 8, 14, 18, 22) are particularly helpful in discrete wavelet packet transformations. These filters aim to minimize the difference between a valid scaling filter and the ideal sine lowpass filter. In this work, the researcher will use the fk8 wavelet.

3. WAVELET SHRINKAGE

Wavelet shrinking, often known as "wave shrink," is a crucial step following WT assessment to remove noise from observations. The researchers discovered that the frequency of noisy coefficients created after transformation is lower than that of the initial observation coefficients. By applying a frequency threshold that cancels out noise features while keeping the original observation coefficients, shrinkage is a regularly used technique to reduce risk or noise. As a test, it produces a list of significant transformation coefficients that satisfy the threshold cut-off. This is the simpler basic nonlinear improvement of wavelet coefficients according to Donoho and Johnston. If a coefficient's absolute value is less than the chosen threshold's cut-off level, the coefficient is regarded as zero. To show the observation X_t with noise, use the following formula:

$$X_t = S_t + n_t, \qquad t = 0,1,2,\dots,N-1 \qquad (11)$$

Where: S_t is a noise-free real signal, n_t are noise or independent normal random variables that must be inferred from the noisy observation Xt (Leavline et al., 2011). The following stages provide an overview of the wavelet reduction technique. DWT transforms the observed time series into wavelet space. The specified shrinkage function and a defined threshold value are used to decrease and modify wavelet coefficients. The inverse DWT is applied to wavelet coefficients, resulting in a smoothed signal with reduced noise (Kozłowski, 2005).

3.1. Selection of Threshold

Choosing the correct threshold is an important step in the denoising process. Excessively high thresholds may prevent the removal of noisy components. Setting the threshold too low might result in a smoothed signal that loses its features (Raimondo, 2002). The threshold value should be both high enough to remove noise and low enough to preserve the signal's important properties. Therefore, the threshold must be properly determined. Wavelet coefficient thresholds may be determined using many bases, including Universal, Minimax, Rigorous SURE, and Heuristic SURE. The paper will focus on the SURE threshold because to its importance in the practical portion. (Nason, 1996).

3.2. Rules for Thresholding

WT coefficients allow for various thresholding processes, such as soft, hard, firm, mid, and non-negative garrote thresholds. The applied side will focus on soft rule-based thresholding. Donoho and Johnstone introduced the Soft Threshold approach for wavelet coefficients (Wn), an extension of the popular Hard Threshold method for reducing noise. It may be described as follows:

$$W_n^{(ST)} = Sign(W_n)(|W_n| - \delta)_+ \qquad (12)$$

here,

$$Sign\ (W_n) = \begin{bmatrix} +1 & if\ W_n > 0 \\ 0 & if\ W_n = 0 \\ -1 & if\ W_n < 0 \end{bmatrix} \qquad (13)$$

Also, we have:

$$(|W_n| - \delta)_+ = \begin{bmatrix} (|W_n| - \delta) & if\ (|W_n| - \delta) \geq 0 \\ 0 & if\ Otherwise \end{bmatrix} \qquad (14)$$

The soft threshold rule applies the "terminate or delete" or "shrink or kill" rule, resulting in zero termination of values below the threshold and preservation of values above the threshold. As a result, it performs a continuous function (Gençay and Whitcher, 2001).

3. APPLICATION ASPECT

The general budget can be clarified through the time series (1997-2021) of general expenditures (x1) as the first variable, and general revenues (x2) as the second variable in the general budget of the Kurdistan Region, which is summarized in the Appendix. There is a general trend in the time series of general revenues and expenditures, as shown in the following Figure 1.

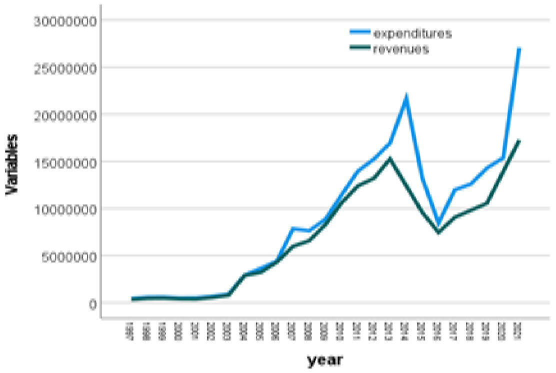

Figure 1: Time series for the general expenditures and revenues

The linear correlation coefficient between general expenditures and revenues amounted to 97%, which is positive and significant because the p-value is equal to zero, which is less than the level of significance (0.05). Figure 2 explains the cross-correlation between general expenditures (X1) and revenues (X2).

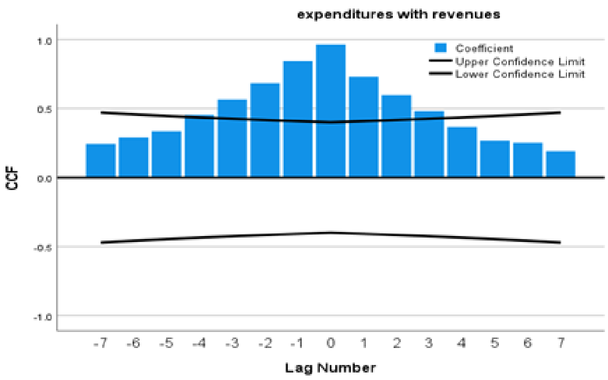

Figure 2:Cross-Correlation between (x1) and (x2)

To confirm whether the data followed a normal distribution or not, the Kolmogorov-Smirnov test was used on the two variables of the study. The result of the analysis showed that the p-value for the first variable was **(**0.66**)** and for the second variable was **(**0.52**)**, and both values are less than the significance level value of 0.05, which indicates that the data follows a normal distribution.

The purpose of the Augmented Dickey-Fuller Test (ADF) is to ascertain if a time series is stable because of the unit root or not close to a mean or linear trend. It tests the following hypothesis:

Null Hypothesis: The data contains a unit root

Alternative Hypothesis: The data does not contain a unit root

Table 1 displays the ADF stationarity test findings for the original time series, as well as the first and second (Diff(.)) differences. The ADF test results in Table 1 showing that the time series of general expenditures is stationary at a second difference. The absolute test statistic (3.7161) is larger than the critical value (1.9507) and the p-value (0.001) is less than the significance level (0.05). The time series of general revenues is stationary at the first difference, as the absolute test statistic (2.3411) exceeds the critical value (1.9513) and the p-value (0.0215) is smaller than the significance threshold (0.05).

Table 1: Augmented Dickey-Fuller Test

Data	Test Statistics	p-value	Critical Value	Null Rejected
X_1	0.6273	0.8439	-1.9507	false
Diff (X_1)	-1.6336	0.0944	-1.9513	false
DiffDiff (X_1)	**-3.7161**	**0.0010**	-1.9518	**true**
X_2	1.0109	0.9120	-1.9507	false
Diff (X_2)	**-2.3411**	**0.0215**	-1.9513	**true**
DiffDiff (X_2)	**-5.0212**	**0.0010**	-1.9518	**true**

Figures 3 and 4 show the sample Autocorrelation and Partial Autocorrelation Function for variables (X1) and (X2), respectively.

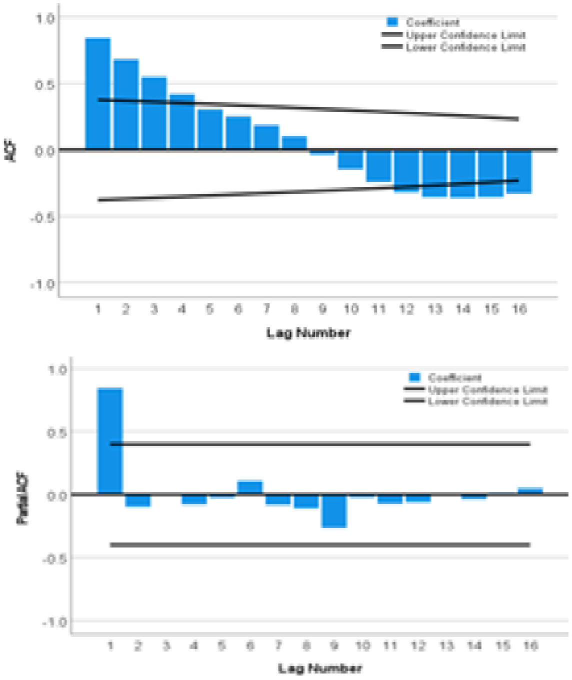

Figure 3:ACF, and PACF for the series X1

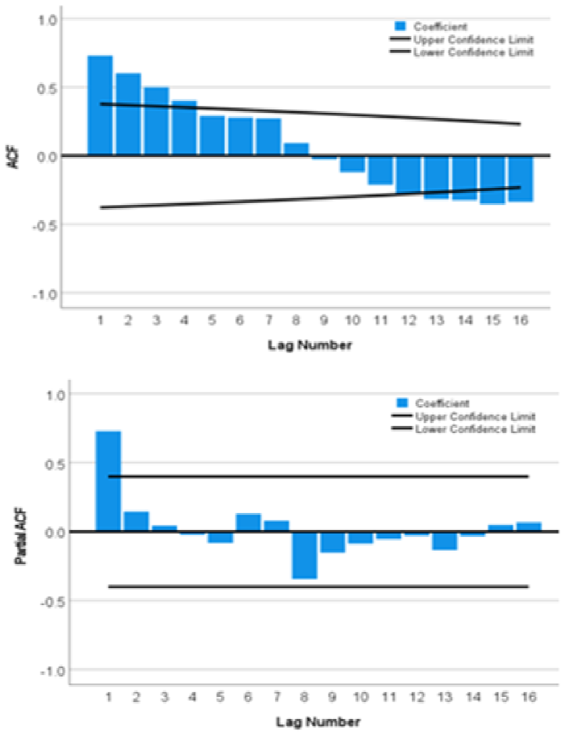

Figure 4: ACF, and PACF for the series X2

3.1. Classical VAR Models for original time series data

VAR models are an extension of the AR model involving two variables to capitalize on cross-correlation. Now we will look for a model that can predict both X1 and X2. The AR-stationary 2-2-dimensional VAR model with a linear temporal trend (VAR) was chosen based on the lowest AIC and BIC values among 12 feasible models. The vector autoregressive model has the following equation:

$$(1-\phi_1 L-\phi_2 L^2)y_t = \delta t + \varepsilon_t \qquad (15)$$

The estimation results for the classical VAR (2) model can be shown in Table 2.

Table 2: Classical Estimation Results

Parameter	Value	Standard Error	t Statistic	p-Value
AR {1} (1,1)	-0.40179	0.325	-1.236	0.217
AR {1} (2,1)	-0.44588	0.220	-2.023	0.043
AR {1} (1,2)	2.1347	0.293	7.293	3.0253e-13
AR {1} (2,2)	1.4092	0.198	7.101	1.2428e-12
AR {2} (1,1)	-0.046762	0.294	-0.159	0.873
AR {2} (2,1)	-0.27641	0.199	-1.388	0.165
AR {2} (1,2)	-0.57395	0.670	-0.860	0.390
AR {2} (2,2)	0.136	0.452	0.301	0.764
Trend (1)	211558.559	96609.818	2.190	0.029
Trend (2)	255336.879	65506.809	3.898	9.7043e-05

Table 2 clearly shows the statistical significance of some estimated parameters, the trend 1 and 2 parameters, which supports the strength of this model. Figure 5 shows the model fit for both variables.

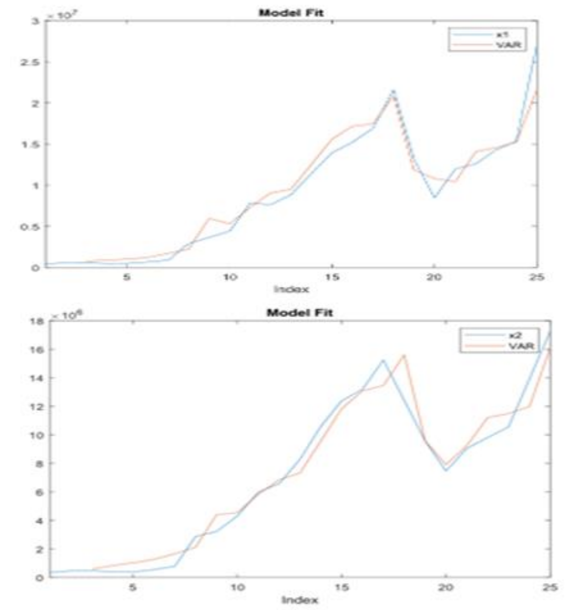

Figure 5:Model fit for the classical model VAR (2)

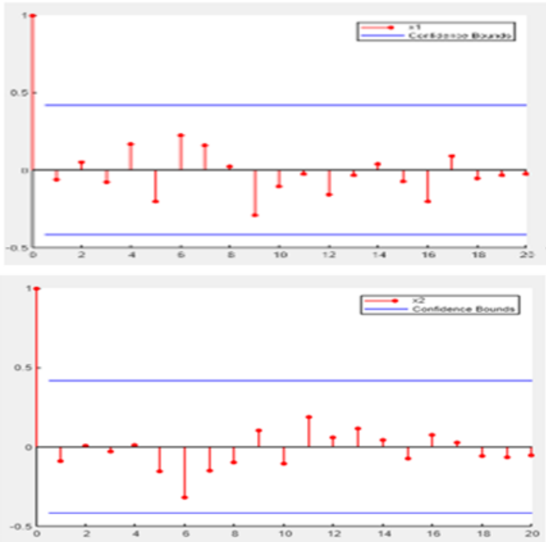

Figure 6: Residual Sample Autocorrelation Function for classical model VAR (2)

Figure 6 illustrates how the residuals change around the zero line. Additionally, both variables' autocorrelation coefficients are entirely within the confidence range. Furthermore, most residual values are inside the standard curve. The VAR (2) model has passed all relevant tests and can anticipate future values.

3.2. Proposed VAR Models for time series data:

The suggested technique involves employing wavelets in MATLAB to decrease noise and contamination in time series data, followed by forecasting after reconstructing the VAR (2) model. Wavelets such as Coiflets, Daubechies, Symlets, and Fejér-Korovkin were utilized alongside thresholding methods like Sure-Threshold and a soft rule. Figure 7 depicts the wavelet Coif2 using the Sure-Threshold approach and soft rules.

The time series data (1 and 2 represent X1 and X2, respectively) was transformed using wavelet filters, de-noised, and then reconstructed using the VAR (2) model.

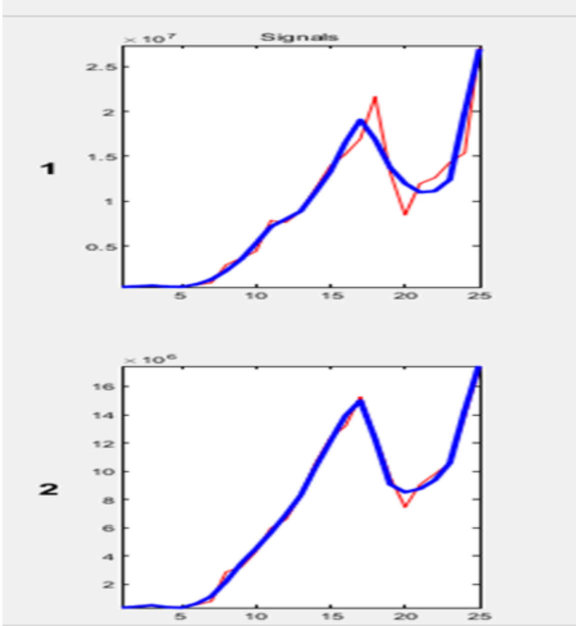

Figure 7: Coif2 bivariate wavelet

Table 3 displays estimate results for the proposed Coif2 VAR (2) model.

Table 3: Proposed Estimation Results (Coif2)

Parameter	Value	Standard Error	t Statistic	p-value
AR {1} (1,1)	0.26953	0.343	0.785	0.433
AR {1} (2,1)	-0.20465	0.251	-0.816	0.414
AR {1} (1,2)	2.1552	0.261	8.243	1.6821e-16
AR {1} (2,2)	1.5109	0.191	7.916	2.4457e-15
AR {2} (1,1)	0.49388	0.282	1.753	0.080
AR {2} (2,1)	-0.12855	0.206	-0.625	0.532
AR {2} (1,2)	-1.9746	0.643	-3.069	0.002
AR {2} (2,2)	-0.41361	0.470	-0.881	0.379
Trend (1)	105333.3485	83215.545	1.266	0.206
Trend (2)	218722.6195	60746.189	3.6006	0.0003

Table 3 demonstrates the statistical significance of calculated parameters (trend 1 and 2), highlighting the model's strength. Figure 8 illustrates the model fit for both variables.

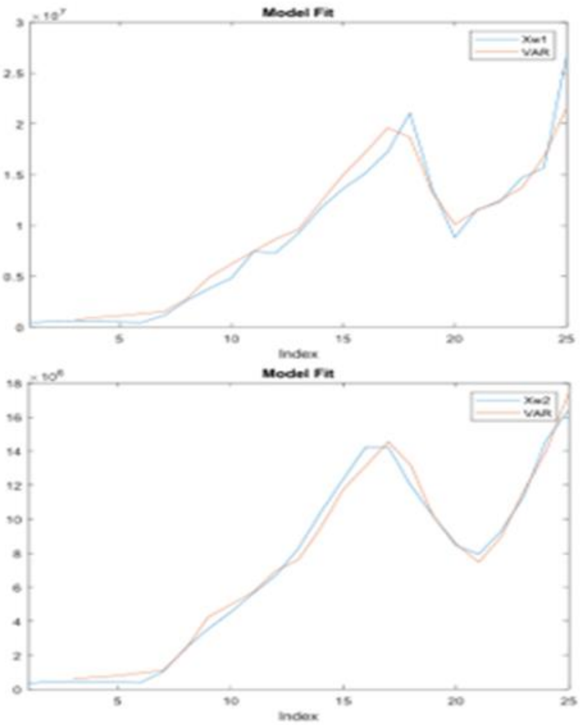

Figure 8:Model fit for the proposed Coif2 model VAR (2)

Figure 9 illustrates how the residuals change around the zero line. Additionally, both variables' autocorrelation coefficients are entirely within the confidence range. Furthermore, most residual values are inside the standard curve. The VAR (2) model has passed all relevant tests and can anticipate future values.

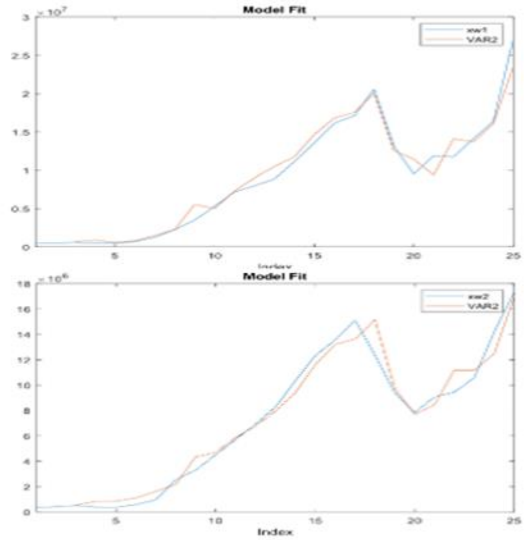

Figure 9: Residual Sample Autocorrelation Function for proposed Coif2 model VAR (2)

Daubechies by order (4) at the same time some thresholding methods were used, with the use of a soft rule, and then reconstructing the VAR (2) model. Table 4 shows the estimation results for the second proposed (db4) VAR (2) model.

Table 4: Proposed Estimation Results (db4)

Parameter	Value	Standard Error	t Statistic	p-value
AR {1} (1,1)	-0.195	0.248	-0.789	0.430
AR {1} (2,1)	0.113	0.092	1.224	0.221
AR {1} (1,2)	1.771	0.288	6.148	0.000
AR {1} (2,2)	1.550	0.107	14.430	0.000
AR {2} (1,1)	-0.440	0.224	-1.959	0.050
AR {2} (2,1)	-0.047	0.084	-0.563	0.574
AR {2} (1,2)	-0.013	0.496	-0.027	0.979
AR {2} (2,2)	-0.902	0.185	-4.872	0.000
Trend (1)	230944.965	88469.135	2.611	0.009
Trend (2)	168810.336	32989.574	5.117	0.000

Table 4 clearly shows the statistical significance of some estimated parameters, the trend 1 and 2 parameters, which supports the strength of this model. Figure 10 shows the model fit for both variables.

Figure 11 illustrates how the residuals change around the zero line. Additionally, both variables' autocorrelation coefficients are entirely within the confidence range. Furthermore, most residual values are inside the standard curve. The VAR (2) model has passed all relevant tests and can anticipate future values.

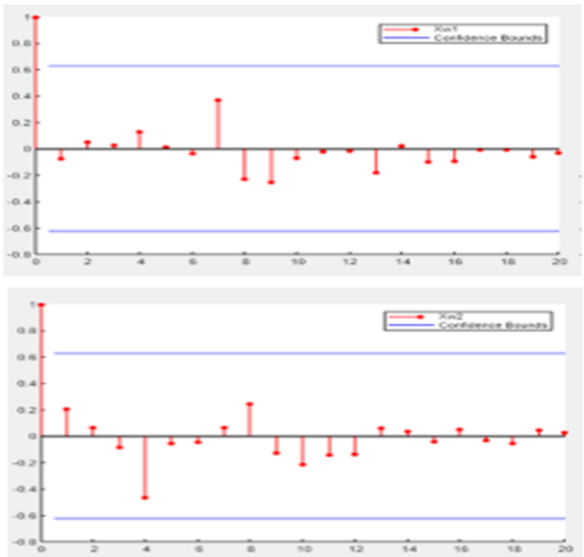

Figure 10:Model fit for the proposed (db4) model VAR (2)

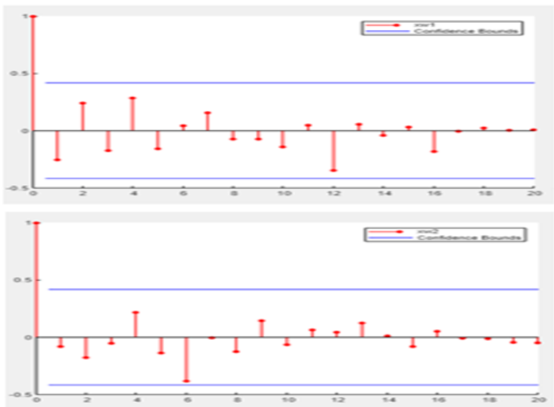

Figure 11: Residual Sample Autocorrelation Function for proposed db4 model VAR (2)

Symlets by order (3) at the same time some thresholding methods were used, with the use of a soft rule, and then reconstructing the VAR (2) model. Table 5 shows the estimation results for the third proposed (Sym3) VAR (2) model.

Table 5: Proposed Estimation Results (Sym3)

Parameter	Value	Standard Error	t Statistic	p-value
AR {1} (1,1)	-0.242	0.263	-0.919	0.358
AR {1} (2,1)	-0.154	0.110	-1.397	0.162
AR {1} (1,2)	1.580	0.334	4.724	0.000
AR {1} (2,2)	1.500	0.140	10.705	0.000
AR {2} (1,1)	-0.663	0.266	-2.488	0.013
AR {2} (2,1)	-0.249	0.112	-2.232	0.026
AR {2} (1,2)	0.460	0.546	0.842	0.400
AR {2} (2,2)	-0.311	0.229	-1.358	0.174
Trend (1)	272528.730	89959.415	3.030	0.002
Trend (2)	209441.534	37699.609	5.556	0.000

Table 5 demonstrates the statistical significance of calculated parameters (trend 1 and 2), highlighting the model's strength. Figure 12 displays the model fit for both variables.

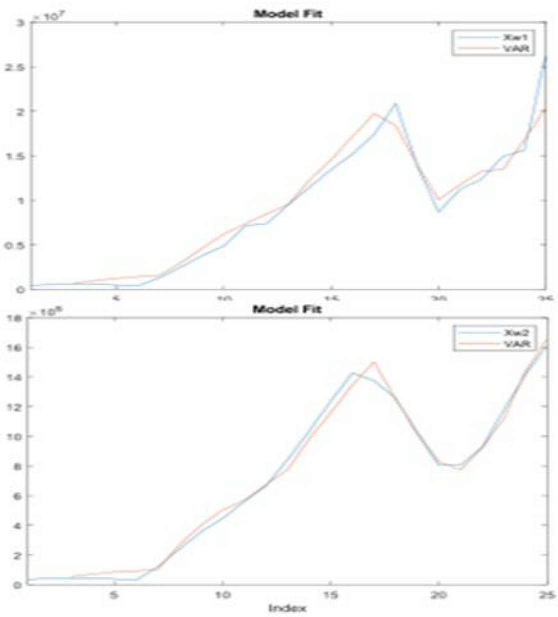

Figure 12: Model fit for the proposed (Sym3) model VAR (2)

Figure 13 illustrates how the residuals change around the zero line. Additionally, both variables' autocorrelation coefficients are entirely within the confidence range. Furthermore, most residual values are inside the standard curve. The VAR (2) model has passed all relevant tests and can anticipate future values.

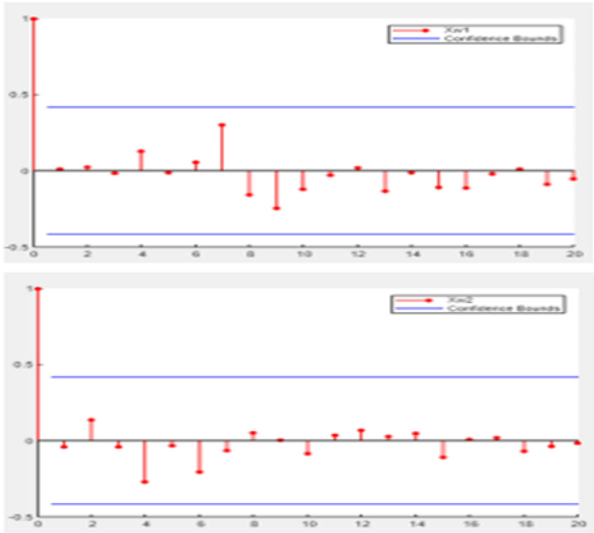

Figure 13: Residual Sample Autocorrelation Function for proposed Sym3 model VAR (2)

Fejér–Korovkin by order (8) at the same time some thresholding methods were used, with the use of a soft rule, and then reconstructing the VAR (2) model. Table 6 shows the estimation results for the fourth proposed (FK8) VAR (2) model.

Table 6:Proposed Estimation Results (FK8)

Parameter	Value	Standard Error	t Statistic	P-Value
AR {1} (1,1)	-0.343	0.280	-1.227	0.220
AR {1} (2,1)	-0.176	0.082	-2.145	0.032
AR {1} (1,2)	1.579	0.391	4.043	0.000
AR {1} (2,2)	1.731	0.115	15.092	0.000
AR {2} (1,1)	-0.614	0.309	-1.989	0.047
AR {2} (2,1)	-0.031	0.091	-0.340	0.734
AR {2} (1,2)	0.543	0.628	0.864	0.387
AR {2} (2,2)	-0.743	0.185	-4.024	0.000
Trend (1)	267447.707	98609.915	2.712	0.007
Trend (2)	174915.839	28961.007	6.040	0.000

Table 6 clearly shows the statistical significance of some estimated parameters, the trend 1 and 2 parameters, which supports the strength of this model. Figure 14 shows the model fit for both variables.

Figure 15 illustrates how the residuals change around the zero line. Additionally, both variables' autocorrelation coefficients are entirely within the confidence range. Furthermore, most residual values are inside the standard curve. The VAR (2) model has passed all relevant tests and can anticipate future values.

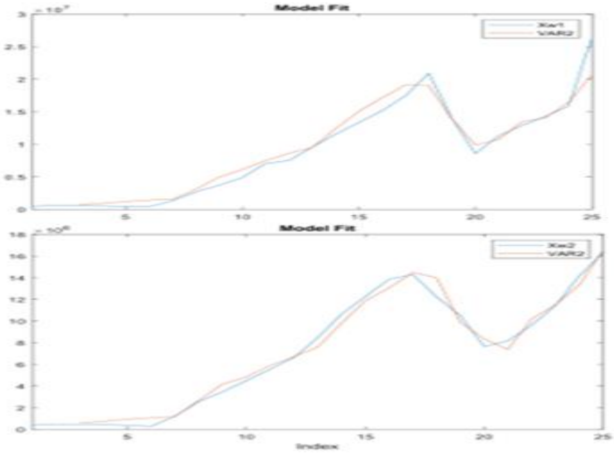

Figure 14:Model fit for the proposed (FK8) model VAR (2)

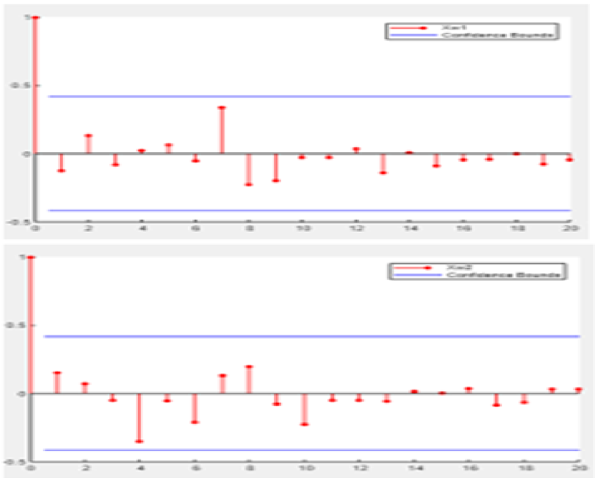

Figure 15: Residual Sample Autocorrelation Function for

3.3. Best Model Choosing

The best model is selected by comparing the three estimated models based on criteria AIC and BIC as shown in Table 7:

Table 7:VAR (2) models' efficiency criteria

Method	AIC	BIC
The classical	1447.076	1458.431
The first proposed (Coif2)	1431.330	1442.685
The second proposed (db4)	**1409.924**	**1421.278**
The third proposed (Sym3)	1421.738	1433.093
The fourth proposed (FK8)	1412.818	1424.172

Table 7 shows that the suggested method of the second one (db4) was the optimal and the best because the values of criterion AIC and BIC are less than their value in the other methods, so the following second proposed model was relied upon:

$$\left(1-\begin{bmatrix}-0.19539 & 1.77110\\ 0.11309 & 1.55010\end{bmatrix}L-\begin{bmatrix}-0.43953 & -0.01334\\ -0.04707 & -0.90171\end{bmatrix}L^2\right)\begin{pmatrix}y_{1t}\\ y_{2t}\end{pmatrix}=\begin{pmatrix}230944.9645\\ 168810.3355\end{pmatrix}$$

3.4. Forecasting the General Expenditures and Revenues

The best model estimated above VAR (2) with db4 was used to forecast the general expenditures and revenues for the Kurdistan Region of Iraq for the five years (2022-2026), and are summarized in Table 8:

Table 8. Forecasting the general expenditures and revenues VAR (2) with db4

Year	General Expenditures	General Revenues
2022	22327000	18581000
2023	22379000	19485000
2024	26081000	19319000
2025	25258000	18830000
2026	23159000	18124000

Table 8 shows that there is an expected increase in the coming years in general expenditures and revenues, with the balance of general expenditures remaining higher than general revenues, which constitutes the continuation of the general deficit in the budget of the Kurdistan Region of Iraq in the coming years, as shown in Figure 16.

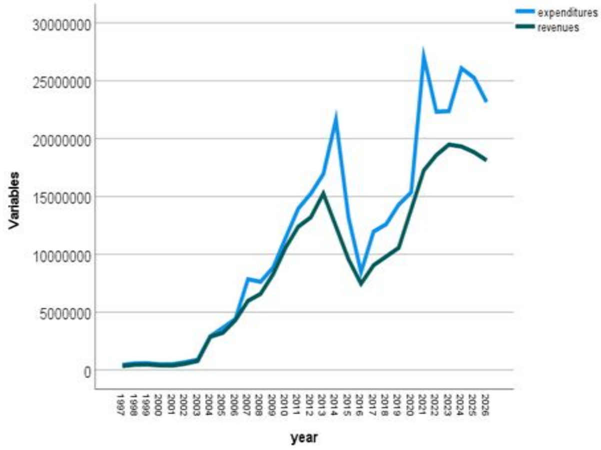

Figure 16: The time series data with forecasting

CONCLUSION & RECOMMENDATIONS

Through the study of simulation and real data, the following main conclusions and recommendations were reached:

Conclusions

1. All the proposed methods are better than the classical method for this data depending on criteria AIC and BIC.
2. The second presented method (db4) was the optimal one for the VAR (2) model of general expenditures and revenues.
3. There is a simple linear correlation between general revenues and expenditures amounting to 97%, which is positive and significant.
4. The forecast for the period (2022-2026) shows an increase in general expenditures and revenues.
5. There is a general trend in the time series of general expenditures and revenues indicating a significant increase in the sustainable deficit of the general budget in the Kurdistan Region of Iraq.

Recommendations

1. Approval of the proposed estimated model and forecasting values for the coming years to draw plans.
2. Develop financial and economic stability in the Kurdistan Region-Iraq to achieve financial sustainability and reduce the general budget deficit.
3. Conducting a prospective study based on another time series analysis using Wavelet Shrinkage for general revenues and expenditures data.
4. Conducting a prospective study based on VAR time series analysis using another Wavelet type for general revenues and expenditures data.

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APPENDIX

General budget in the Kurdistan Region of Iraq

Year	General revenues (x1)	General expenditures (x2)
1997	347402	444761
1998	463157	572680
1999	480637	585192
2000	408142	491947
2001	391861	506545
2002	548184	678204
2003	789956	896089
2004	2878345	2914617
2005	3222007	3651075
2006	4315702	4418818
2007	5982356	7847660
2008	6580784	7628783
2009	8283172	8857263
2010	10597176	11432176
2011	12386000	13950670
2012	13200950	15245797
2013	15257849	16942749
2014	12417303	21636133
2015	9576758	13203700
2016	7462207	8467041

2017	9057407	11965562
2018	9810288	12602532
2019	10563169	14294089
2020	13911778	15368197
2021	17260387	27066987