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A NEW CONJUGATE GRADIENT METHOD BASED ON LOGISTIC MAPPING FOR UNCONSTRAINED OPTIMIZATION AND ITS APPLICATION IN REGRESSION ANALYSIS

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ABSTRACT:
The study tackles the critical need for efficient optimization techniques in unconstrained optimization problems, where conventional techniques often suffer from slow and inefficient convergence. There is still a need for algorithms that strike a balance between computational efficiency and robustness, despite advancements in gradient-based techniques. This work introduces a novel conjugate gradient algorithm based on the logistic mapping formula. As part of the methodology, descent conditions are established, and the suggested algorithm's global convergence properties are thoroughly examined. Comprehensive numerical experiments are used for empirical validation, and the new algorithm is compared to the Polak-Ribière-Polyak (PRP) algorithm. The suggested approach performs better than the PR algorithm, according to the results, and is more efficient since it needs fewer function evaluations and iterations to reach convergence. Furthermore, the usefulness of the suggested approach is demonstrated by its actual use in regression analysis, notably in the modelling of population estimates for the Kurdistan Region of Iraq. In contrast to conventional least squares techniques, the method maintains low relative error rates while producing accurate predictions. All things considered, this study presents the novel conjugate gradient algorithm as an effective tool for handling challenging optimisation problems in both theoretical and real-world contexts.
KEYWORDS: optimization; conjugate gradient; step size; regression analysis.

1. INTRODUCTION

Unconstrained optimization issues entail minimizing an objective function that relies exclusively on the real variables, devoid of any restrictions on the variables' values. This can be expressed mathematically as:

$$\min f(x) \qquad \forall\, x \in R^n, \tag{1.1}$$

where $f : R^n \to R$, $f \in C^1$, and $g_k = \nabla f(x_k)$ is a gradient at point x_k. The conjugate gradient method (CG) is an optimization algorithm that lies between the steepest descent method and the Newton method in terms of computational complexity and convergence behaviour. Unlike the steepest descent method, which updates the solution solely in the direction of the negative gradient, the (CG) method modifies this direction by incorporating a positive linear combination of the previous search direction. This adjustment helps to overcome the steepest descent method's limitation of slow convergence. The (CG) method only requires the computation of first-order derivatives, specifically the gradient of the objective function, which significantly reduces computational cost compared to methods that require second-order derivatives, such as the Newton method. The Newton method involves calculating and inverting the Hessian matrix, a process that is computationally expensive and often impractical for large-scale problems. In contrast, a key advantage of the (CG) method is that it does not require the Hessian matrix or its approximation, making it particularly well-suited for large-scale optimization challenges.
The (CG) algorithm typically generates a sequence x_k as:

$$x_{k+1} = x_k + \alpha_k d_k, \; k = 0,1,2,\dots, \tag{1.2}$$

where $\alpha_k > 0$ is the step length. d_k is a search direction given by:
$d_0 = -g_0$, and

$$d_{k+1} = -g_{k+1} + \beta_k d_k \; for\; k \geq 1, \tag{1.3}$$

the parameter β_k is crucial, with different choices leading to various CG methods. Over the years, numerous variants of this scheme have been proposed and widely applied in practice, such as the Fletcher-Reeves (FR), (Fletcher & Reeves, 1964), Polak-Ribière-Polyak (PRP), (Polak & Ribiere, 1969; Polyak B. T., 1969), Hestenes-Stiefel (HS), (Hestenes & Stiefel, 1952), Liu-Storey (LS), (Liu & Storey, 1991), Dai-Yuan (DY), (Dai & Yuan, 1999), and Conjugate-Descent (CD), (Fletcher, 1987)methods.

$$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}, \qquad \text{Fletcher-Reeves (1964).}$$

$$\beta_k^{PRP} = \frac{g_{k+1}^T y_k}{g_k^T g_k}, \qquad \text{Polak-Ribière-Polyak (1969).}$$

$$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k}, \qquad \text{Hestenes-Stiefel (1952).}$$

$$\beta_k^{LS} = \frac{g_{k+1}^T y_k}{-d_k^T g_k} \qquad \text{Lia-Storey (1991).}$$

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$\boldsymbol{\beta_k^{DY}} = \frac{g_{k+1}^T g_{k+1}}{d_k^T y_k}$, Dia-Yuan (1999).

$\boldsymbol{\beta_k^{CD}} = -\frac{g_{k+1}^T g_{k+1}}{d_k^T g_k}$, Conjugate Descent (1987).

Numerous researchers have dedicated their efforts to refining (CG) methods, motivated by their widespread adoption in solving optimization problems, as well as their properties of global convergence and low memory utilization. Interestingly, most of these efforts are directed towards improving conventional CG methods, which are the first generation of CG algorithms. For more details, see (Ibrahim & Mohammed, 2022, 2024; Ibrahim & Shareef, 2019; Jahwar et al., 2024; Shareef & Ibrahim, 2016).

Since classical CG methods have well-established convergence properties and fundamental principles, they serve as the basis for all subsequent variants. In spite of their success, these methods have stimulated a wealth of research studies focused on particular issues, like enhancing robustness, scalability, and convergence rates for high-dimensional problems. Since the gradients in these approaches are mutually orthogonal and the parameters β_k are similar, they are equivalent when f is a highly convex quadratic function and the line search is exact. However, their behaviour differs significantly when applied to broad nonlinear functions with imprecise line searches. Although the strong convergent features of FR, DY, and CD algorithms are well known, jamming may cause them to perform poorly in real-world scenarios. Furthermore, PRP, HS, and LS approaches often outperform one another even if they might not converge in general.

Naturally, researchers strive to develop new techniques that combine the best features of these two categories. Numerous hybrid approaches have been proposed thus far. For instance, Touati-Ahmed and Storey (Touati-Ahmed & Storey, 1990) originally introduced a hybrid conjugate gradient algorithm that combines the FR and PRP techniques in 1990. Subsequently, Hu and Storey (Hu & Storey, 1991), along with Gilbert and Nocedal (Gilbert & Nocedal, 1992), explored other hybrid systems related to the PRP and FR techniques. Dai and Yuan (Dai & Yuan, 2001) combined the DY technique with the HS method to create two hybrid CG algorithm aimed at enhancing the practical application of the DY method. For large-scale problems in unconstrained optimization, Andrei (Andrei, 2008) presented a novel hybrid CG approach, referred to as the HYBRID algorithm, which is based on the HS and DY methods. A primary characteristic of this hybrid method is that the search direction is the Newton direction. Remarkably, this hybrid technique often out performs certain complex conjugate gradient methods in various applications.

This research focuses on how new improvements in conjugate gradient methods can contribute to enhanced performance of optimization algorithms. Specifically, how can techniques such as logistic mapping enhance these methods.

The objective of this study is to develop a new conjugate gradient method incorporating logistic mapping and to analyze its performance compared to traditional methods. The focus is on improving convergence rates and robustness, particularly in applications related to regression analysis.

The motivation behind this study is to address the need for more efficient optimization techniques capable of handling large-scale problems and real-world scenarios effectively.

The challenges include ensuring the robustness of the new method, addressing convergence issues, and validating its effectiveness in practical applications.

This study highlights how logistic mapping can improve conjugate gradient methods, offering new insights into optimization by enhancing convergence rates and expanding practical applications.

The structure of the paper is as follows: In next section, we present our particular technique and several methods we used to determine the parameter β_k. Under appropriate conditions, the descent and adequate descent properties of the suggested approach are also covered with the global convergence. In Section 3, preliminary numerical data are shown. We create a summary of our article in the end.

2. NEW CONJUGATE GRADIENT METHODS

This section presents a new (CG) method for solving (1.1). The CG parameter of the algorithm is based on the logistic mapping formula (Lu et al., 2006), which is widely used in optimization. By utilizing the logistic mapping along with the CG parameter from Polak-Ribière-Polyak (PRP), the algorithm's performance can be enhanced., From logistic mapping formula we have

$$\beta_k^{New} = \beta_k^{PRP}(1 - \beta_k^{PRP}), \tag{2.1}$$

to achieve balance, multiplying the second term of the equation (2.1), by scalar we get:

$$\beta_k^{New} = \left(\frac{g_{k+1}^T y_k}{g_k^T g_k} - \rho\left(\frac{g_{k+1}^T y_k}{g_k^T g_k}\right)^2\right), \tag{2.2}$$

where $\rho = \mu \frac{g_k^T g_k}{d_k^T y_k}$ and $0 < \mu \leq 1$.

After some algebraic operations, we get:

$$\beta_k^{New} = \frac{g_{k+1}^T y_k}{g_k^T g_k} - \mu\left(\frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k}\right). \tag{2.3}$$

1.1 Algorithm of the New (CG) Method:

Step (1): Initialization

Begin with an initial point $x_0 \in R^n$.

Step (2): Initialization and Gradient Computation

Set $k = 0$ and compute the gradient $g_0 = \nabla f(x_0)$.

Define the search direction $d_0 = -g_0$.

If $g_0 = 0$, terminate the algorithm.

Step (3): Line Search

Determine the step length α_k , to minimize the objective function $f(x_{k+1})$ by using cubic line search

Step (5): Update

Update the iterate: $x_{k+1} = x_k + \alpha_k d_k$

Step (6): Gradient Update and Termination Check

Compute $g_{k+1} = \nabla f(x_{k+1})$, If: $\|g_{k+1}\| \leq 10^{-5}$, then stop

Step (7): Conjugacy Update

Calculate β_k based on a specific rule (2.3)

Step (8): Conjugate Direction Update

Update the search direction:

$$d_{k+1} = -g_{k+1} + \beta_k^{New} d_k,$$

Step (9): Convergence Check

If $k = n$ or if $|g_k^T g_{k+1}| \geq 0.2\|g_{k+1}\|^2$ return to

Step 2; otherwise, increment k and return to Step 3

Theorem 1. Let $\{x_k\}$ and $\{d_k\}$, be two sequences generated by new method. Then the d_{k+1} satisfies the:

$$d_{k+1}^T g_{k+1} \leq 0.$$

Proof: Using (1.3) and (2.3), we've obtained

$$d_{k+1} = -g_{k+1} + (\frac{g_{k+1}^T y_k}{g_k^T g_k} - \mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k}) d_k, \quad (2.4)$$

by multiplying the aforementioned equation by g_{k+1}^T, on both sides, we obtain:

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k}{g_k^T g_k} g_{k+1}^T d_k - \mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k} g_{k+1}^T d_k \,, \quad (2.5)$$

when the α_k is determined through an exact line search, yielding $d_k^T g_{k+1} = 0$, the resultant equation

$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 \leq 0.$

the descent condition validated.

In the case of an inexact line search, where $d_k^T g_{k+1} \neq 0$. Since we have $g_{k+1}^T d_k < d_k^T y_k$. So, we get

$$g_{k+1}^T d_{k+1} < -\|g_{k+1}\|^2 + \beta_k^{PRP} g_{k+1}^T d_k - \mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k}, \quad (2.6)$$

The affirmation that the initial two terms of equation (2.6) adhere to non-positivity stems from the fulfilment of the descent condition by the Polak-Ribière-Polyak (PRP) algorithm. This condition is a fundamental criterion in optimization theory, ensuring that iterative algorithms make progress towards minimizing the objective function.

Since the search direction of (PRP) method represented the descent condition as

$.-\|g_{k+1}\|^2 + \frac{(g_{k+1}^T y_k)(g_{k+1}^T d_k)}{g_k^T g_k} \leq 0,$

It signifies that the chosen descent directions align with the objective of function minimization, thereby facilitating convergence. This condition serves as a crucial safeguard against divergent behaviour and ensures the convergence trajectory of the algorithm.

The adherence of the PRP algorithm to the descent condition engenders confidence in subsequent analytical derivations, particularly in equation (2.6). By establishing the negativity of the initial terms, the proof establishes a robust foundation for affirming the convergence of the CG method, even in scenarios involving inexact line search methodologies.

And it is obviously that μ, $g_k^T g_k$ and $(g_{k+1}^T y_k)^2$ are positive, so, we get to the third term of equation (2.6), which is less than or equal to zero. Hence, we get

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{PR} g_{k+1}^T d_k - \mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k} g_{k+1}^T d_k \leq 0.$$

The descent condition is proved.

Theorem 2. Let $\{x_k\}$ and $\{d_k\}$ be two sequences generated by new method. Then the d_{k+1} satisfies the sufficient descent condition:

$g_{k+1}^T d_{k+1} \leq -c\|g_{k+1}\|^2$, for any $k \geq 0$.

Proof: The two initial terms of equation (2.6) are demonstrably less than or equal to zero due to the properties of the Polak-Ribière-Polyak (PRP) algorithm, which achieve the descent condition. Therefore, we obtain:

$$g_{k+1}^T d_{k+1} < -(\mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k \|g_{k+1}\|^2}) \|g_{k+1}\|^2,$$

let $c = \mu \frac{(g_{k+1}^T y_k)^2}{g_k^T g_k \|g_{k+1}\|^2}$.

Then, $g_{k+1}^T d_{k+1} \leq -c\|g_{k+1}\|^2$.■

In this section, we establish the global convergence of the new method by relying on the following basic assumptions regarding the objective function.

Assumption (*): (Zoutendijk, 1970),

1. **Lower Bound:** The objective function $f(x)$ is bounded below on R^n . The level set $\delta = \{x \,|f(x) \leq f(x_0)\}$ is bounded.
2. **Continuity and Differentiability:** The objective function $f : R^n \to R$ is continuously differentiable on R^n.
3. **Gradient Bound**: In some neighbourhood N of δ, f is continuously differentiable, and its gradient is Lipschitz continuous with Lipschitz constant $\delta > 0$, i.e.

$\|g(x) - g(y)\| \leq \delta\|x - y\| \;\; \forall\, x, y \in \delta.$

From the above assumptions, that there exists a positive constant b such that

$$\|g(x)\| \leq b \;\; \forall x \in \delta. \quad (2.7)$$

If f is a uniformly convex, $\exists\, \vartheta > 0$ such that:

$$\left(g(x) - g(y)\right)^T (x - y) \geq \vartheta\|x - y\|^2 \in \Omega, \quad (2.8)$$

we can rewrite the above equation in the following manner:

$$y_k^T v_k \geq \vartheta\|v_k\|^2, \quad (2.9)$$

These assumptions are standard in optimization theory and provide a foundation for demonstrating the convergence properties of the proposed method. Specifically, the continuity and differentiability assumption ensure the smoothness of the objective function, while the Lipschitz condition on the gradient guarantees that changes in the gradient are controlled. Finally, the lower bound condition ensures that the optimization process does not diverge to $-\infty$, thus supporting the argument for global convergence.

Lemma 1. (Zhang et al., 2006). Assuming the aforementioned conditions hold. Consider the methods (1.2) and (1.3), where d_{k+1} is a descent direction and α_k satisfies the standard Wolfe line search. If

$\sum_{k\geq 1} \frac{1}{\|d_{k+1}\|^2} = \infty$,

then, $\lim_{k\to\infty} inf \;\|g_{k+1}\| = 0$.

Theorem 3. If the sequences $\{x_k\}, \{d_k\}, \{g_k\}, \{\alpha_k\}$, are created by our algorithm, the assumptions (*) hold, the following properties can be established:

$$\lim_{k\to\infty} inf \;\|g_{k+1}\| = 0 .$$

Proof: From equations (1.3) and (2.3), we have

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \left|\frac{g_{k+1}^T y_k}{g_k^T g_k} - \mu(\frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k})\right| \| d_k\| , \quad (2.10)$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \left(\left|\frac{g_{k+1}^T y_k}{g_k^T g_k}\right| + \mu\left|\frac{(g_{k+1}^T y_k)^2}{g_k^T g_k d_k^T y_k}\right|\right) \| d_k\| ,$$

since,

$$g_{k+1}^T y_k \leq \|\, g_{k+1}\|\|\, y_k\| , \quad (2.11)$$

from Lipschitz Condition $\|y_k\| \leq L\|v_k\|$, and by using (2.9), we get

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \left(\frac{\|g_{k+1}\|\|\, y_k\|}{\|g_k\|^2} + \frac{(\|\, g_{k+1}\|\|\, y_k\|)^2}{\|g_k\|^2\vartheta\|v_k\|^2}\right) \| d_k\|,$$

$$\|d_{k+1}\| \leq \|g_{k+1}\| + \left(\frac{Lb\|v_k\|}{\|g_k\|^2} + \frac{L^2 b^2}{\|g_k\|^2\vartheta}\right) \| d_k\|,$$

Since, $\|v_k\| = \|x - x_k\|$, $D = max\{\|x - x_k\|\}, \forall\, x, x_k \in R\}$.

Hence,

$$\|d_{k+1}\| \leq b + \left(\frac{LbD}{\|g_k\|^2} + \frac{L^2 b^2}{\|g_k\|^2\vartheta}\right) \frac{D}{\alpha} = \beta ,$$

$$\Rightarrow \sum_{k\geq 1} \frac{1}{\|d_{k+1}\|^2} \geq \sum_{k\geq 1} \frac{1}{\beta^2} = \infty,$$

$$\Rightarrow \sum_{k\geq 1} \frac{1}{\|d_{k+1}\|^2} = \infty ,$$

By using lemma 1, we get $\lim_{k\to\infty} inf\|g_k\| = 0$ .The proof complete.

3. NUMERICAL RESULTS

This section presents the numerical results of the new method for unconstrained optimization and its application to regression analysis.

3.1 The Unconstrained Optimization

The objective of this subsection is to assess the efficacy of our novel method in addressing optimization challenges, juxtaposed against the Polak-Ribière-Polyak (PRP), method. We employ a comparative analysis, utilizing well-established nonlinear problems characterized by varying dimensionalities (where $(10 \leq n \leq 5000)$). Each computational implementation is meticulously crafted in FORTRAN 95 to ensure precision and reliability. Central to our evaluation is the utilization of the cubic interpolation method within the line search procedure. This method leverages both function and gradient variables to navigate the optimization landscape effectively. Key performance metrics, namely the (NOI) and the (NOF), are meticulously documented and explicitly presented in the results table (Table 1). The experimental results, as delineated in Table 2, underscore the superiority of our proposed technique over the PRP method. This superiority is evident in terms of both NOF and NOI, reaffirming the efficacy and efficiency of our novel approach in comparison to the established PRP method. This comprehensive evaluation framework not only provides empirical validation of our method's effectiveness but also underscores its potential to advance the state-of-the-art in optimization methodologies.

Table 1: The results of the new method compared with the Polak-Ribière-Polyak (PRP) method.

Test Function	n	PRP		NEW	
		NOI	NOF	NOI	NOF
Wolfe	5	14	29	12	20
	10	32	65	25	50
	100	49	99	42	80
	500	58	117	48	90
	1000	64	129	48	90
	5000	99	214	83	161
Mile	5	37	116	29	89
	10	37	116	29	89
	100	44	148	29	89
	500	44	148	29	89
	1000	50	180	50	180
	5000	50	180	50	180
Central	5	22	159	22	159
	10	22	159	22	159
	100	22	159	22	159
	500	23	171	23	170
	1000	23	171	23	170
	5000	30	270	22	159
Powell	5	40	120	30	80
	10	40	120	32	90
	100	43	135	32	90
	500	46	150	32	90
	1000	46	150	38	97
	5000	50	180	38	97
Sum	5	6	39	6	39
	10	6	34	6	34
	100	14	80	11	59
	500	21	123	17	86
	1000	23	127	17	86
	5000	31	145	25	120
Wood	5	29	67	29	67
	10	29	67	29	67
	100	30	69	30	69
	500	30	69	30	69
	1000	30	69	30	69
	5000	30	69	30	69
Cubic	4	15	45	14	39
	10	16	47	14	39
	100	16	47	14	39
	500	16	47	14	39
	1000	16	47	15	43
	5000	16	47	15	43
Rosen	4	30	85	28	65
	10	30	85	28	65
	100	30	85	28	65
	500	30	85	28	65
	1000	30	85	28	65
	5000	30	85	28	65
Total		**1539**	**5233**	**1324**	**4193**

Table 2: The improvement percentage of the new method compared with the Polak-Ribière-Polyak (PRP) method.

Tools	PRP	New
NOI	100%	86.02989 %
NOF	100%	80.12612268 %

3.2 Application of the New Method to Regression Analysis

Regression analysis is a crucial statistical method frequently utilized in areas such as accounting, economics, management, physics, finance, and beyond (Christensen, 1996; Vandeginste, 1989). It is employed to examine the relationship between independent and dependent variables within different datasets. The purpose of regression analysis can be outlined as follows:

$y = h(x_1, x_2, \dots, x_p + \varepsilon),$

where $x_i, i = 1,2, \dots p, p > 0$ is the predictor, y is the response variable, and ε is the error. The linear regression function is derived such that

$y = a_0 + a_1x_1 + a_2x_2 + \cdots + a_px_p + \varepsilon.$

This method is typically applied when the relationship between x and y can be represented by a straight line, although such cases are rare. As a result, nonlinear regression models are often employed. This paper focuses on the nonlinear regression approach.

This subsection provides a detailed overview of population estimates for the Kurdistan Region of Iraq (KRI) from 1965 to 2020. The statistics in Table 3 are sourced from the data collected by the Kurdistan Region Statistics Office, Ministry of Planning, Kurdistan Regional Government (Kurdistan Region Statistics Office, Ministry of Planning, 2021). In this analysis, the years of data collection are represented by the x-variable, while the population figures for KRI serve as the y-variable. Data from 1965 to 2014 will be used for fitting the model, while the data from 2020 will be reserved for error analysis.

Table 3: Population estimates of KRI.

Years	KRI Population
1965	902,000
1987	2,015,466
1997	2,861,701
2014	5,332,600
2020	6,171,083

The approximate function for the nonlinear least squares method is derived using the data in above Table as follows:

$$f(x) = 339358.2x^2 - 282987.7x + 940224.2. \qquad (3.1)$$

The function (3.1) is use to approximate the value of y based on value of x from 2014 - 2017. Let x_j denotes number of years and y_j be the recorded cases of drug addicts. Then, the above least squares method (3.1) is transformed into the following unconstrained minimization problems:

$$\min_{x\in R^n} f(x) = \sum_{j=1}^{n} \left(\left(u_0 + u_1 x_j + u_2 x_j^2\right) - y_j \right)^2, \qquad (3.2)$$

Data from 1965 to 2014 are used to develop the nonlinear quadratic model using the least squares method, along with the associated test function for the unconstrained optimization problem. It is evident from this analysis that the x_j and the value of y_j values exhibit a parabolic relationship, as described by the regression function defined in (3.2) with the regression parameters u_0, u_1 and u_2.

$$\min_{x\in R^2} \sum_{j=1}^{n} E_j^2 = \sum_{j=1}^{n} \left(\left(u_0 + u_1 x + u_2 x^2\right) - y_j \right)^2$$

However, the data for 2020 is excluded from the unconstrained optimization function to facilitate the calculation of relative errors for the predicted data. Consequently, the proposed method is used to solve the test function (3.2) utilizing the strong Wolfe line search technique, with the results presented in Table 4.

Table 4: Test results for optimization of quadratic model using new method.

Initial Points	No. for Iteration	CPU Time
(1,1,1)	13	0.015291
(2,2,2)	13	0.028205
(10,10,10)	13	0.015354
(15,15,15)	15	0.023973

Table 5 displays the relative error of the new method compared to the least squares method. A smaller relative error value indicates greater accuracy and a better fit to the observed dataset.

Table 5: Estimation point and relative errors for 2020 data.

Models	Point	Relative Error
New	6195047.5029	0.0039
Least Square	6375777.5441	0.0332

CONCLUSION

This work introduces a novel (CG) method that based on the logistic mapping formula to enhance optimization problem-solving. The proposed method satisfies both descent and sufficient descent conditions, with the latter being a stronger criterion that significantly improves numerical performance. A comprehensive analysis of the global convergence properties confirms the method's effectiveness, establishing it as a robust approach in the optimization filed. Numerical experiments demonstrate that the new method outperforms the traditional Polak-Ribière-Polyak CG method, particularly in terms of Numbers of Iterations (NOI) and Numbers of Function evaluations (NOF). The results indicate that the proposed algorithm achieves superior convergence rates and requires fewer computational resources, making it especially valuable for large-scale problems with dimensions ranging from 10 to 5000. Furthermore, the application of the new method to regression analysis, specifically in modelling population estimates for the Kurdistan Region of Iraq, reveals its practical utility. The model produced accurate predictions while maintaining low relative error rates compared to traditional least squares methods. Future research may explore additional refinements and broader applications to further enhance its effectiveness in complex optimization challenges.

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