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ADOMIAN DECOMPOSITION METHOD AND VARIATIONAL ITERATION METHOD FOR SOLVING SASA-SATSUMA EQUATION

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ABSTRACT:

The Sasa-Satsuma equation is an integrable higher-order nonlinear Schrodinger equation. In this paper, two schemes are proposed to study numerical solutions of the Sasa-Satsuma nonlinear Schrödinger equation with initial conditions using the Adomian decomposition method and the variational iteration method. Both approaches produce quickly convergent series for each scheme with particularly important features. The present results have been displayed graphically and, in a table, to demonstrate the effectiveness and applicability of those techniques. The results obtained by the Adomian decomposition method are compared with the exact solution as well as the results obtained by variational iteration method. A comparison between the two approaches reveals that the Adomian decomposition approach is closer and more efficient than the variational iteration approach.

KEYWORDS: Sasa-Satsuma Equation, Nonlinear Schrödinger Equation, Numerical Solution, Adomian Decomposition Method, Variational Iteration Method.

1. INTRODUCTION

Due to its fundamental structure in soliton transmission technology, the optical soliton pulse has significant uses in the telecommunications and optical fiber industries (Agrawal & Kivshar, 2003). According to the nonlinear Schrödinger equation (NLSE), which describes the fundamental events, the nonlinear form of the refractive index is directly proportional to the light intensity (Hamasalh, Murad, & Ismael, 2024). Furthermore, only the self-phase modulation effects and group velocity dispersion are taken into account in the NLSE (Adams & Hughes, 2019). However, it is important to consider the effects of stimulated Raman scattering, self-steepening, and third-order dispersion for ultrashort pulses in optical fibers (J. P. Wu & Geng, 2017). Owing to these effects, the Sasa-Satsuma equation (SSE) can be used to describe the dynamics of the ultrashort pulses.

One of the nontrivial integrable extensions of the NLSE is the SSE (C. Wu *et al.*, 2022). While looking for integrable examples of a higher-order NLSE that Kodama and Hasegawa had proposed, Sasa and Satsuma came across it (Sasa & Satsuma, 1991). Due to its huge use, it received a lot of attention and has been thoroughly researched (Arnous , Hamasalh, Murad, *et al.*, 2024; Li *et al.*, 2020;). Thus, the purpose of this article is to analyses the numerical solution of the issue using the variation iteration method (VIM) and the Adomian decomposition method (ADM) and show how each approach affects the equation. Semi-analytic techniques are the key concept behind those approaches. Additionally, as the new methods do not require the discretization of variables, computation round-off errors are not introduced, and large computer memory and time are not required. However, the ADM considers the approximate solution to be an infinite series, which typically converges to the exact solution (Dehghan & Salehi, 2011). Moreover, a comparison between the exact solution and the numerical approaches will be conducted.

Adomian first proposed the decomposition technique at the start of the 1980s (Tatari *et al.*, 2011). The ADM offers an efficient method for the analytical solution of a broad and extensive class of dynamical systems that represent actual physical issues (Bera & Ray, 2005; Kumar & Malik, 2023). The ADM is a well-known systematic approach for the practical solution of operator equations that are linear or nonlinear, deterministic or stochastic, such as integral equations, integro-differential equations, ordinary differential equations, and partial differential equations (Azzo & Manaa, 2022; Duan *et al.*, 2012; Sabali *et al.*, 2021). In order to construct the numerical solution of the differential equations, the ADM has been successfully used as Kuramoto Sivashinsky Equation (Manaa *et al.*, 2014), the Bagley–Torvik equation (Bera & Ray, 2005), coupled Burger's equations (An & Chen, 2008), Time-Fractional Coupled Klein-Gordon Schrodinger Equation (Fotros & Hesameddini, 2012), Fokker–Planck equation (Tatari *et al.*, 2007), and so also (Sabali *et al.*, 2018). It offers an effective numerical solution with few computations and can be used to find approximation or even closed-form analytical solutions of differential equations without linearization or perturbation (An & Chen, 2008; Hesameddini & Fotros, 2012).

Chinese mathematician Ji-Huan He proposed the VIM, which has recently been the subject of extensive research by

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engineers and scientists and has been successfully applied to a variety of nonlinear issues, like the Duffing equation (He, 1999), the Schrödinger equation (Sadigh Behzadi, 2011), the self-focusing NLSE (Wazwaz & Kaur, 2019), and the nonlinear fractional differential equation (Odibat & Momani, 2006). Its simplicity and ease of use are advantages, and it is based on the Lagrange multiplier (Al-Bar, &Sweilam , 2007). The approach has been demonstrated to solve a wide class of nonlinear problems efficiently, simply, and precisely. Usually, very accurate solutions are obtained after one or two iterations (Akram Abdulqadr & Nawzad Mustafa, 2025; Ghorbani & Saberi-Nadjafi, 2009).

2. MATHEMATICAL MODEL

The formulation of SSE in its dimensionless form is as follows:

$$i\psi_t + \frac{1}{2}\psi_{xx} + |\psi|^2\psi + i\alpha(\psi_{xxx} + 6|\psi|^2\psi_x + 3(|\psi|^2)_x\psi) = 0. \quad (1)$$

where $i = \sqrt{-1}$.

The symbol for time in dimensionless form is t, and the non-dimensional distance is represented by x. $\psi(x,t)$, often known as the soliton profile, is the dependent variable with complex values. For the integrable nonlinear Schrödinger perturbation, the positive real parameter α is used. The exact soliton solution of equation (1), as in Akhmediev *et al.* (2015), is:

$$\psi(x,t) = \zeta tanh(\sigma x + \gamma t)e^{i(\theta x + \lambda t)}, \quad (2)$$

and the initial condition is:

$$\psi(x,0) = \zeta tanh(\sigma x)e^{i\theta x}. \quad (3)$$

Where $\zeta = \frac{-i\sqrt{\omega^2(1+6\theta\alpha-\omega\alpha)}}{2\sqrt{1+6\theta\alpha-2\omega\alpha}}$, $\sigma = \frac{\omega}{2}$, $\gamma = \frac{\eta}{2}$ and $\alpha, \omega, \eta, \theta \in \mathbb{R}$.

3. ADOMIAN DECOMPOSITION METHOD

In this section, the fundamental concept of the ADM and the derivative of it on the SSE will be introduced.

Basic Idea of the Adomian Decomposition Method:

Consider the following nonlinear differential equation:

$$\ell(\psi(x,t)) = \Re(\psi) + \aleph(\psi). \quad (4)$$

The operator of the higher-order derivatives is denoted by ℓ, $\aleph(\psi)$ is a nonlinear term and $\Re(\psi)$ a linear term. Under the initial conditions:

$$\psi(x,0) = \hbar(x). \quad (5)$$

Using the specified condition and applying the inverse operator $\ell_t^{-1} = \int_0^t (.)dt$ to both sides of equation (4), we get

$$\psi(x,t) = \hbar(x) + \ell_t^{-1}(\Re(\psi)) + \ell_t^{-1}(\aleph(\psi)). \quad (6)$$

The sum of the generic solution to the following equation is

$$\psi(x,t) = \sum_{n=0}^{\infty} \psi_n. \quad (7)$$

Also, $\aleph(\psi)$ which is usually represented by the sum of series, and it is the nonlinear operator

$$\aleph(\psi) = \sum_{n=0}^{\infty} A_n. \quad (8)$$

Where A_n is Adomian's polynomial of $\psi_0(x)$, $\psi_1(x)$, $\psi_2(x)$,... which is defined as

$$A_n = \frac{1}{n!}\left(\frac{d^n}{d\lambda^n}\left(\aleph\left(\sum_{i=0}^{n}\lambda^i\psi_i\right)\right)\right)_{\lambda=0}. \quad (9)$$

The following recursive relations are now available to us by the ADM:

$$\psi_0(x,t) = \hbar(x). \quad (10)$$

$$\psi_{n+1}(x,t) = \ell_t^{-1}(\Re(\psi_n)) + \ell_t^{-1}(\aleph(\psi_n)). \quad (11)$$

Derivative of Adomian Decomposition Method for Sasa-Satsuma Equation:

Consider the SSE (1) in an operator form:

$$i\ell_t(\psi) + \frac{1}{2}\ell_{xx}(\psi) + |\psi|^2\psi + i\alpha(\ell_{xxx}(\psi) + 6|\psi|^2\ell_x(\psi) + 3\psi\ell_x(|\psi|^2)) = 0. \quad (12)$$

Where, $\ell_t = \frac{\partial}{\partial t}$, $\ell_x = \frac{\partial}{\partial x}$, $\ell_{xx} = \frac{\partial^2}{\partial x^2}$, and $\ell_{xxx} = \frac{\partial^3}{\partial x^3}$.

Applying the inverse operator ℓ_t^{-1} to both sides of the equation (12) and using the initial condition (5), yields

$$\psi(x,t) = \hbar(x) + \frac{1}{2}i\ell_t^{-1}(\ell_{xx}(\psi)) + i\ell_t^{-1}(|\psi|^2\psi) - \alpha\left(\ell_t^{-1}(\ell_{xxx}(\psi)) + 6\ell_t^{-1}(|\psi|^2\ell_x(\psi)) + 3\ell_t^{-1}(\psi\ell_x(|\psi|^2))\right). \quad (13)$$

Usually, the solutions $\psi(x,t)$ defined as an infinite series

$$\sum_{n=0}^{\infty}\psi_n = \hbar(x) + \frac{1}{2}i\ell_t^{-1}\left(\ell_{xx}\left(\sum_{n=0}^{\infty}\psi_n\right)\right) + i\ell_t^{-1}\left(\sum_{n=0}^{\infty}A_n\right) - \alpha\left(\ell_t^{-1}\left(\ell_{xxx}\left(\sum_{n=0}^{\infty}\psi_n\right)\right) + 6\ell_t^{-1}\left(\sum_{n=0}^{\infty}B_n\right) + 3\ell_t^{-1}\left(\sum_{n=0}^{\infty}C_n\right)\right). \quad (14)$$

Then, we get

$$\psi_0(x,t) = \hbar(x), \quad (15)$$

$$\psi_{n+1}(x,t) = \frac{1}{2}i\ell_t^{-1}(\ell_{xx}(\psi_n)) + i\ell_t^{-1}(A_n) - \alpha\left(\ell_t^{-1}(\ell_{xxx}(\psi_n)) + 6\ell_t^{-1}(B_n) + 3\ell_t^{-1}(C_n)\right). \quad (16)$$

Where $A = \aleph(\psi) = \psi|\psi|^2$, $B = \aleph(\psi) = |\psi|^2\frac{\partial\psi}{\partial x}$, and $C = \aleph(\psi) = \psi\frac{\partial|\psi|^2}{\partial x}$, are the Adomian polynomial which is defined by (9), and can be de rived by

$A_0 = \psi_0^2\overline{\psi_0}$, $A_1 = \psi_0^2\overline{\psi_1} + 2\psi_0\overline{\psi_0}\psi_1$, ...

$B_0 = \psi_0\overline{\psi_0}\frac{\partial\psi_1}{\partial x}$, $B_1 = \psi_0\overline{\psi_0}\frac{\partial\psi_1}{\partial x} + \psi_0\overline{\psi_1}\frac{\partial\psi_0}{\partial x} + \psi_1\overline{\psi_0}\frac{\partial\psi_0}{\partial x}, \ldots$

$C_0 = \psi_0\frac{\partial}{\partial x}(\psi_0\overline{\psi_0})$, $C_1 = \psi_1\frac{\partial}{\partial x}(\psi_0\overline{\psi_0}) + \psi_0\frac{\partial}{\partial x}(\psi_0\overline{\psi_1}) + \psi_0\frac{\partial}{\partial x}(\psi_1\overline{\psi_0})$.

Similar methods can be used to construct other polynomials. The first few elements of $\psi_n(x,t)$ appear right away after establishing

$$\psi_0 = \hbar(x), \tag{17}$$

$$\psi_1 = \frac{1}{2}i\ell_t^{-1}\big(\ell_{xx}(\psi_0)\big) + i\ell_t^{-1}(A_0) - \alpha\left(\ell_t^{-1}\big(\ell_{xxx}(\psi_0)\big) + 6\ell_t^{-1}(B_0) + 3\ell_t^{-1}(C_0)\right), \tag{18}$$

$$\psi_2 = \frac{1}{2}i\ell_t^{-1}\big(\ell_{xx}(\psi_1)\big) + i\ell_t^{-1}(A_1) - \alpha\left(\ell_t^{-1}\big(\ell_{xxx}(\psi_1)\big) + 6\ell_t^{-1}(B_1) + 3\ell_t^{-1}(C_1)\right). \tag{19}$$

Then

$$\psi(x,t) = \sum_{n=0}^{\infty}\psi_n = \psi_0(x,t) + \psi_1(x,t) + \psi_2(x,t) + \cdots. \tag{20}$$

VARIATIONAL ITERATION METHOD

This section aims to introduce the fundamental concept of the VIM and its application to the SSE.

Basic Idea of the Variational Iteration Method:

Examine a nonlinear differential equation in its generic form:

$$\ell\big(\psi(x,t)\big) = \Re(\psi) + \aleph(\psi). \tag{21}$$

The VIM permits the application of a correction functional like

$$\psi_{n+1}(x,t) = \psi_n(x,t) + \int_0^t \lambda(\tau)\left(\ell\big(\psi(x,t)\big) - \Re(\psi) - \aleph(\psi)\right)d\tau. \tag{22}$$

where λ is a general Lagrange multiplier. Be aware that the Lagrange multiplier λ can be either a function or a constant and that $\tilde{\psi}_n$ is a limited value, meaning it acts like a constant, this leads to $\delta\tilde{\psi}_n = 0$, where δ is the variational derivative. It has been demonstrated in the literature that the variational theory provides the best way to identify the Lagrange multiplier λ. The successive approximations $\psi_{n+1}(x,t), n \geq 0$ of the solution $\psi(x,t)$ can be easily obtained using the resulting Lagrange multiplier and any function ψ_0. Consequently, the remedy is

$$\psi(x,t) = \lim_{n\to\infty}\psi_n(x,t). \tag{23}$$

Derivative of Variational Iteration Method for Sasa-Satsuma Equation:

Consider the SSE (1), The function for rectification is supplied by

$$\psi_{n+1}(x,t) = \psi_n(x,t) + \int_0^t \lambda(\tau)\left(\frac{\partial\psi_n}{\partial\tau} - i\frac{1}{2}\frac{\partial^2\psi_n}{\partial x^2} - i|\psi_n|^2\psi_n + \alpha\left(\frac{\partial^3\psi_n}{\partial x^3} + 6|\psi_n|^2\frac{\partial\psi_n}{\partial x} + 3\psi_n\frac{\partial|\psi_n|^2}{\partial x}\right)\right)d\tau. \tag{24}$$

where we used $\lambda(\tau) = -1$. Adding to the correction function equation (24)

$$\psi_{n+1}(x,t) = \psi_n(x,t) - \int_0^t \frac{\partial\psi_n}{\partial\tau} - i\frac{1}{2}\frac{\partial^2\psi_n}{\partial x^2} - i|\psi_n|^2\psi_n + \alpha\left(\frac{\partial^3\psi_n}{\partial x^3} + 6|\psi_n|^2\frac{\partial\psi_n}{\partial x} + 3\psi_n\frac{\partial|\psi_n|^2}{\partial x}\right)d\tau. \tag{25}$$

The initial condition can be used to choose option $\psi_0 = f(x)$. The subsequent approximations obtained by including this selection into the correction function are as follows:

$$\psi_0(x,t) = \hbar(x), \tag{26}$$

$$\psi_1(x,t) = \psi_0(x,t) - \int_0^t \frac{\partial\psi_0}{\partial\tau} - i\frac{1}{2}\frac{\partial^2\psi_0}{\partial x^2} - i|\psi_0|^2\psi_0 + \alpha\left(\frac{\partial^3\psi_0}{\partial x^3} + 6|\psi_0|^2\frac{\partial\psi_0}{\partial x} + 3\psi_0\frac{\partial|\psi_0|^2}{\partial x}\right)d\tau, \tag{27}$$

$$\psi_2(x,t) = \psi_1(x,t) - \int_0^t \frac{\partial\psi_1}{\partial\tau} - i\frac{1}{2}\frac{\partial^2\psi_1}{\partial x^2} - i|\psi_1|^2\psi_1 + \alpha\left(\frac{\partial^3\psi_1}{\partial x^3} + 6|\psi_1|^2\frac{\partial\psi_1}{\partial x} + 3\psi_1\frac{\partial|\psi_1|^2}{\partial x}\right)d\tau. \tag{28}$$

In a similar way, we can find others. Then, the approximate solutions will take the forms:

$$\psi(x,t) = \lim_{n\to\infty}\psi_n(x,t) \approx \psi_n. \tag{29}$$

where n is the closing iteration step.

APPLICATION WITH NUMERICAL RESULT

This section compares the findings of the current research with the exact answer by using the method covered in the previous section to determine the numerical solution of the SSE.

The numerical result of equation (1) when $\eta = 6, \omega = 4, \alpha = 1, \lambda = 3, and\ \theta = 2$ obtained by ADM can be seen below

$$\psi_0 = -\frac{6\sqrt{5}\tanh(2x)\,e^{2xi}\,i}{5}. \tag{30}$$

$$\psi_1 = -\frac{6\sqrt{5}\tanh(2x)\,e^{2xi}\,i}{5} - \frac{12\sqrt{5}\,t\,e^{2xi}}{25}(552\tanh^4(2x)\,i + 308\tanh^3(2x) - 642\tanh^2(2x)\,i - 125\tanh(2x) + 90\,i). \tag{31}$$

$$\psi_2 = \frac{6\sqrt{5}\,e^{2xi}}{125\cosh(2x)^7}\Big(66978\,i\sinh(2x)\,t^2\cosh^6(2x) + 1210440\,t^2\cosh^5(2x) + 492800\,i\sinh(2x)\,t^2\cosh^4(2x) - 4111680\,t^2\cosh^3(2x) - 4611696\,i\sinh(2x)\,t^2\,cosh^2(2x) + 3141600\,t^2\cosh(2x) + 5511168\,i\sinh(2x)\,t^2 - 1830\sinh(2x)\,t\cosh^6(2x) + 4620\,t\cosh^5(2x)\,i + 3080\sinh(2x)\,t\cosh^4(2x) - 5520\,t\cosh^3(2x)\,i - 25\,i\sinh(2x)\cosh^6(2x)\Big). \tag{32}$$

The numerical result of equation (1) obtained by VIM can be seen below

$$\psi_0 = -\frac{6\sqrt{5}\tanh(2x)\,e^{2xi}\,i}{5}. \tag{33}$$

$$\psi_1 = -\frac{6\sqrt{5}\tanh(2x)\,e^{2xi}\,i}{5} - \frac{12\sqrt{5}\,t\,e^{2xi}}{25}(552\tanh^4(2x)\,i + 308\tanh^3(2x) - 642\tanh^2(2x)\,i - 125\tanh(2x) + 90\,i). \tag{34}$$

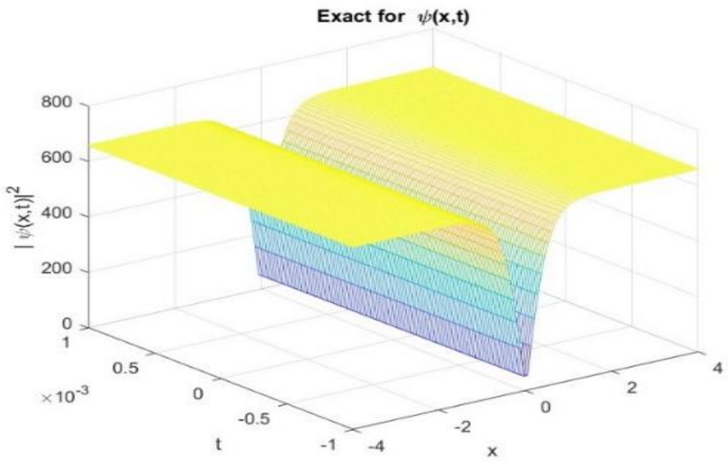
$$\psi_2 = -\frac{6\sqrt{5}\,e^{2xi}}{3125}(1162574954496\,t^4\tanh^{13}(2x)\,i + 700809449472\,t^4\,tanh^{12}(2x) - 4300513191936\,t^4\,tanh^{11}(2x)\,i - 2135661244416\,t^4\,tanh^{10}(2x) + 6284349960192\,t^4\,tanh^{9}(2x)\,i + 2511944593920\,t^4\,tanh^{8}(2x) - 4553215075584\,t^4\,tanh^{7}(2x)\,i - 1398917039616\,t^4\,tanh^{6}(2x) + 1662345013248\,t^4\,tanh^{5}(2x)\,i + 360541279440\,t^4\,tanh^{4}(2x) - 277954213320\,t^4\,tanh^{3}(2x)\,i - 40169206800\,t^4\,tanh^{2}(2x) + 17558791200\,t^4\tanh(2x)\,i + 1452168000\,t^4 + 15795855360\,t^3\tanh^{10}(2x)\,i + 7799362560\,t^3\tanh^{9}(2x) - 44178232320\,t^3\tanh^{8}(2x)\,i - 17265465600\,t^3\tanh^{7}(2x) + 42959669760\,t^3\tanh^{6}(2x)\,i + 12078782400\,t^3\tanh^{5}(2x) - 16543569600\,t^3\tanh^{4}(2x)\,i - 2685620400\,t^3\tanh^{3}(2x) + 2012932800\,t^3\tanh^{2}(2x)i + 161352000\,t^3\tanh(2x) - 46656000\,t^3\,i + 137779200\,t^2\tanh^{7}(2x)\,i + 78540000\,t^2\tanh^{6}(2x) - 298045200\,t^2\tanh^{5}(2x)\,i - 132828000\,t^2\tanh^{4}(2x) + 195072800\,t^2\tanh^{3}(2x)\,i + 60297000\,t^2\tanh^{2}(2x) - 36481250\,t^2\tanh(2x)\,i - 6009000\,t^2 + 138000\,t\tanh^{4}(2x)i + 77000\,t\tanh^{3}(2x) - 160500\,t\tanh^{2}(2x)\,i - 31250\,t\tanh(2x) + 22500\,t\,i + 625\tanh(2x)\,i). \quad (35)$$

Table 1: Exact solution and approximation solution of the methods: ADM, and VIM as $t = 0.0001$ and $-4 \le x \le 4$.

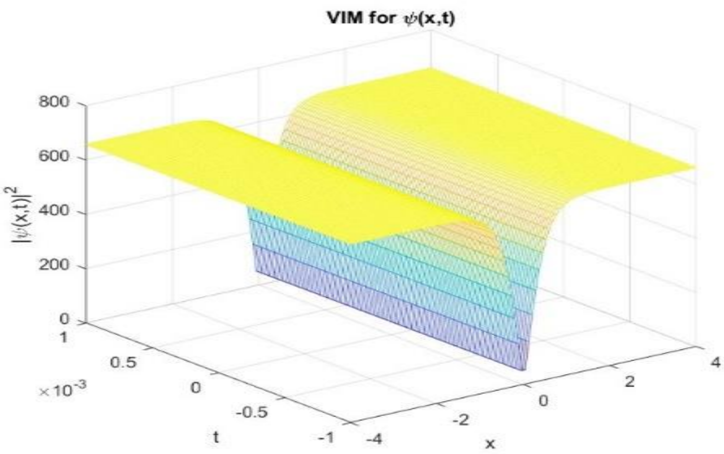
x	Exact	ADM	VIM
-4	7.19999675704251	7.19999687520657	7.19999687147890
-3.1	7.19988131443288	7.19988570020443	7.19988569647741
-2.2	7.19565759196166	7.19581796926491	7.19581796556179
-1.3	7.04276818063409	7.04838142211047	7.04838142057263
-0.4	3.17319894806841	3.20804302110690	3.20804311584250
0.5	4.17756629096868	4.13432907550746	4.13432909999134
1.4	7.09434775438805	7.09042053762901	7.09042053517082
2.3	7.19709239790624	7.19698151480717	7.19698151109463
3.2	7.19992053788026	7.19991750374966	7.19991750002243
4	7.19999676093172	7.19999663559850	7.19999663187084

Table 2: The absolute error of exact solution, ADM, and VIM as $t = 0.0001$ and $-4 \le x \le 4$.

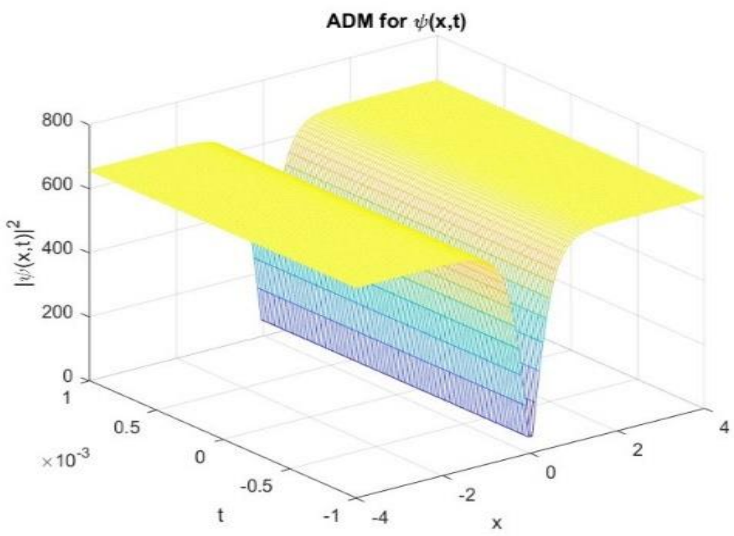
$\lvert\psi ADM - \psi Exact\rvert$	$\lvert\psi VIM - \psi Exact\rvert$
1.18164058982018e-07	1.14436392806283e-07
4.38577155392750e-06	4.38204453612201e-06
1.60377303251025e-04	1.60373600129660e-04
5.61324147637698e-03	5.61323993853691e-03
3.48440730384874e-02	3.48441677740858e-02
4.32372154612253e-02	4.32371909773401e-02
3.92721675904717e-03	3.92721921722838e-03
1.10883099069703e-04	1.10886811606647e-04
3.03413060009916e-06	3.03785782840293e-06
1.25333221134838e-07	1.29060884646037e-07
Total	
$\lvert\psi ADM - \psi Exact\rvert$	$\lvert\psi VIM - \psi Exact\rvert$
8.790067053689166e-02	8.790074171856954e-02



(a) Exact solution of SSE.



(b) ADM of SSE.



(c) VIM of SSE.

Figure 1: The 3D mesh of the exact solution, ADM, and VIM for SSE

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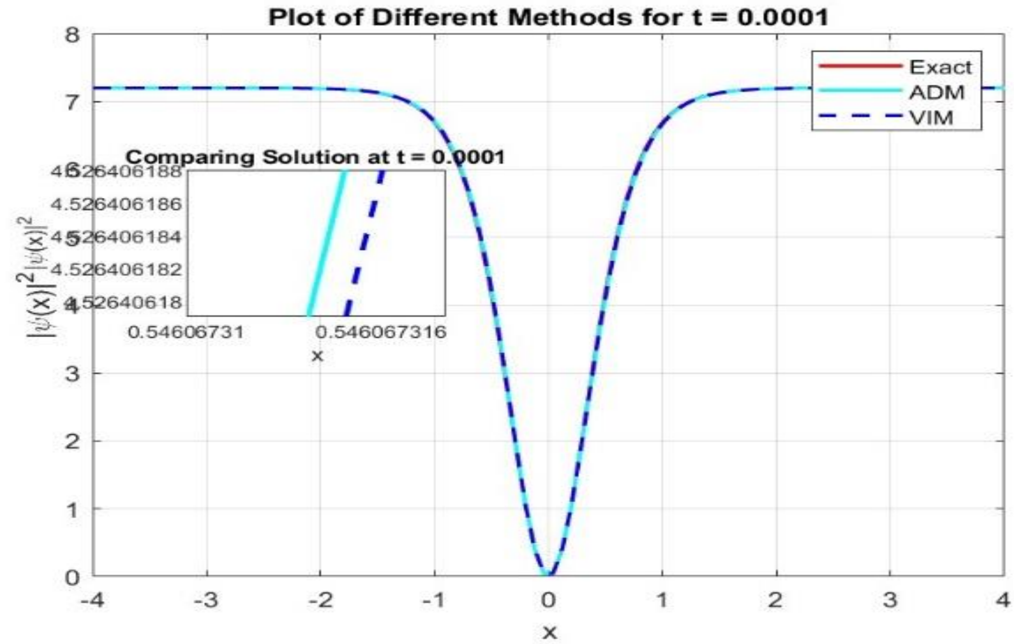
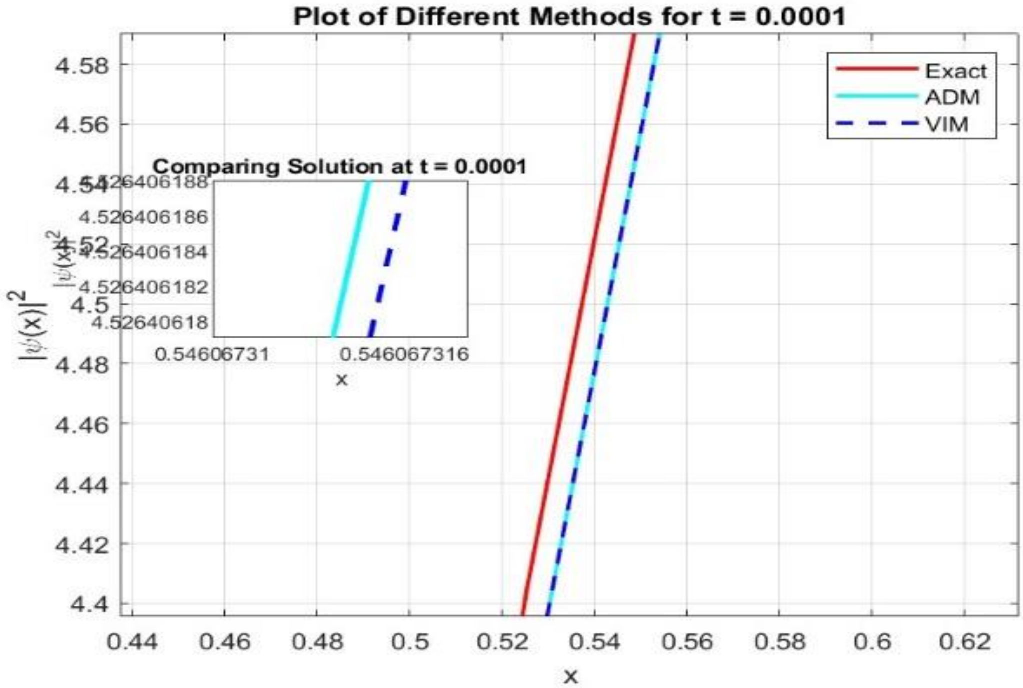


(a) The curves of ADM and VIM to the exact solution.



(b) The zooming curves of ADM and VIM to the exact solution.

Figure 2: The curves show how ADM, and VIM are closer to the exact solution for SSE. While selecting ψ_3 from both methods, when $-4 \leq x \leq 4$ and $t = 0.0001$.

CONCLUSION

In several applications, including optics, higherorder and multicomponent versions of the NLSE are significant. One of these equations, the integrable SSE, has particularly interesting soliton solutions. In this study, the numerical solution of this equation was developed for the first time by both numerical methods, ADM and VIM. Both methods for computing series or exact solutions are significant in applied sciences due to their direct application to all differential and integral equations, reducing computational work while maintaining high numerical accuracy. Both methods demonstrate effectiveness in finding exact solutions to nonlinear problems. Moreover, the VIM requires the evaluation of the Lagrange multiplier, whereas ADM requires the evaluation of the Adomian polynomials that mostly require tedious algebraic calculations. Furthermore, the numerical results that are obtained are compared with the exact solution while choosing ψ_3 from both methods. The results indicate that the absolute error of the ADM is smaller than the VIM, as shown in table (2) and figure (2). As a result, the ADM produces more accurate results compared with the VIM. The high agreement of the approximation of $\psi(x)$ between the methods makes them an alternative for solving the NLSEs. Further work will focus on comparing the results obtained with a new numerical method and those obtained using the current methods.

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Knier A. Salih was responsible for writing the main script and conducting the primary research. Saad A. Manaa supervised, provided guidance, and reviewed the manuscript. Both authors contributed to the final version of the paper.

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Conflict of Interest:

The authors declare that they have no conflict of interest.

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