A NEW CONJUGATE GRADIENT FOR UNCONSTRAINED OPTIMIZATION BASED ON STEP SIZE OF BARZILAI AND BORWEIN

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(Accepted for publication: April 11, 2016)

Abstract:

In this paper, a new formula of β_k is suggested for conjugate gradient method of solving unconstrained optimization problems based on step size of Barzilai and Borwein. Our new proposed CG-method has descent condition, sufficient descent condition and global convergence properties. Numerical comparisons with a standard conjugate gradient algorithm show that this algorithm very effective depending on the number of iterations and the number of functions evaluation.

KeyWords: *unconstrained optimization, conjugate gradient, descent condition, sufficient descent condition,Barzilai and Borwein step size and global convergence.*

1- Introduction

We are concerned with the following unconstrained minimization problem:

$minf(x),\ :\ x \in R^n$ (1.1)

Where $f : R^n \rightarrow R$ is continuously differentiable and its gradient $g_k = \nabla f(x_k)$ is available. There are several kinds of numerical methods for solving (1.1), which include the Steepest Descent (SD) method, the Newton method and Quasi-Newton (QN) methods. Among them, the CG-method is one choice for solving large scale problems, because it does not need any matrices Liu et al.(1993), Liu and Storey(1991)]. CG-methods are iterative methods and at the k-th iteration, it's general form is given by:

$x_{k+1} = x_k + \alpha_k d_k \quad k = 0,1,\ldots$ (1.2)

where $\alpha_k > 0$ is a step size and d_k is the search direction defined by:

$d_{k+1} = -g_{k+1} + \beta_k d_k\ ,\ d_0 = -g_0$ (1.3)

where g_k is the gradient of $f(x)$ at the point x_k. $\beta_k \in R$ is a scalar parameter which characterizes the CG-method. If f is a strictly convex quadratic function and the line search is exact, then the iterative method (1.2)-(1.3) is called linear CG-method. Well-known formulas for β_k are the Fletcher-Reeves (FR) Fletcher and Reeves (1964), Polak-Ribiere- Polyak (PRP) (1969)) , Hestenes-Stiefel (HS) (1952), Dai and Liao (DL) (2001), Conjugate Descent (CD) Fletcher (1987), Liu and Storey (LS) (1991), and Dai and Yuan (DY) (1996), formulas and they are given by:

$\beta_k^{FR} = \frac{g_{k+1}^T g_{k+1}}{g_k^T g_k}$ (1.4)

$\beta_k^{PR} = \frac{g_{k+1}^T y_k}{g_k^T g_k}$ (1.5)

$\beta_k^{HS} = \frac{g_{k+1}^T y_k}{d_k^T y_k}$ (1.6)

$\beta_k^{DL} = \frac{g_{k+1}^T (y_k - t v_k)}{d_k^T y_k}$, where $t > 0$ (1.7)

$\beta_k^{CD} = \frac{g_{k+1}^T g_{k+1}}{-g_k^T d_k}$ (1.8)

$\beta_k^{LS} = \frac{g_{k+1}^T y_k}{-g_k^T d_k}$ (1.9)

$\beta_k^{DY} = \frac{g_{k+1}^T g_{k+1}}{d_k^T y_k}$ (1.10)

Where $\|.\|$ denotes the Euclidean norm, and $y_k = g_{k+1} - g_k$. The global convergence properties of the FR, PRP and HS methods without regular restarts have been studied by many researchers, including Al-Baali(1985) and Gilbert and Nocedal(1992), Zoutendijk (1970), Liu et al (1993),, Powell (1977), and Dai and Yuan(1995),. To establish the convergence results of these methods, it is normally required that the step-length α_k satisfies the following strong Wolfe conditions:

$f(x_k + \alpha_k d_k) - f(x_k) \le \rho \alpha_k g_k^T d_k$ (1.11)

$|g(x_k + \alpha_k d_k)^T d_k| \le -\sigma g_k^T d_k$ (1.12)

Where $\rho \in \left(0, \frac{1}{2}\right)$ and $\sigma \in (0,1)$

Some convergence analysis even require that the step-size α_k can be computed by an exact line search, namely:

$f(x_k + \alpha_k d_k) = \min_{\alpha_k \ge 0} f(x_k + \alpha_k d_k)$ (1.13)

On the other hand, many other numerical methods for unconstrained optimization are proved to be convergent under the standard Wolfe conditions (1.10):

$f(x_k+\alpha_k d_k)-f(x_k)\le \rho\alpha_k g_k^T d_k$(1.14)

$g(x_k+\alpha_k d_k)^T d_k \ge \sigma g_k^T d_k$ (1.15)

For example, see Fletcher(1987),. Hence, it is interesting to investigate whether there exists a CG-method that converges under the standard Wolfe conditions.

In this paper, we present our new formula of$\beta_k$in Section 2. In Section 3 we will proof the descent condition and sufficient descent condition of our new formula. We analyze global convergence of the proposed method with inexact line searches in Section 4. Some interesting numerical results and discussions are presented in Section 5 by comparing our new method with the other CG method. Finally, our conclusions are presented in Section 6.

2- New Conjugate Gradient Algorithm ($\boldsymbol{\beta_k^{New}}$)

In this section, we will derive a new conjugate gradient coefficient for unconstraint optimizations based on $\beta_k^{DL}$by using step size of (Barzilai and Borwein)for finding the minimum of the continuous function $f(x)$.

Consider$v_k = x_{k+1}-x_k=\alpha_k d_k$

Let$v_k^* = \alpha_k^* d_k$ (2.1)

Where $\alpha_k^* = \frac{v_k^T v_k}{v_k^T y_k}$, see [Barzilai and Borwein (1988),]

or

$v_k^* = \frac{v_k^T d_k}{v_k^T y_k} v_k$(2.2)

Now, replacing v_k by v_k^* in (1.7), so, equation (1.7) becomes

$$\beta_k = \frac{g_{k+1}^T\,(y_k - \mathrm{t}\frac{v_k^T d_k}{v_k^T y_k} v_k)}{d_k^T y_k}$$

This implies that

$$\beta_k = \frac{g_{k+1}^T\,(y_k - \mathrm{t}\frac{\alpha_k d_k^T d_k}{d_k^T y_k} d_k)}{d_k^T y_k}$$

After some algebraic operations, we get

$$\beta_k^{New} = \frac{g_{k+1}^T y_k}{d_k^T y_k} - t\,\frac{\alpha_k \|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2}$$

$\beta_k^{New} = \beta_k^{HS} - t\,\frac{\alpha_k \|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2}$(2.3)

Algorithm of New Method ($\boldsymbol{\beta_k^{New}}$):

Step (1): The initial point x_0 , $\varepsilon = 1\times 10^{-5}$.

Step (2): $g_k = \nabla f(x_k)$, If:$g_k = 0$, then stop,

Step (3): set k=0, $d_0 = -g_0$

Step (4): compute $\alpha_k$to minimize $f(x_{k+1})$(i.e.) $f(x_{k+1}) \le f(x_k)$ using cubic line search

Step (5): $x_{k+1} = x_k + \alpha_k d_k$

Step (6): $g_{k+1} = \nabla f(x_{k+1})$, If $\|g_{k+1}\| < \varepsilon$

then stop

Step (7): compute β_k from (2.3)

Step (8): $d_{k+1} = -g_{k+1} + \beta_k^{New} d_k$

Step (9): If $k=n$ or if $|g_k^T g_{k+1}| \le 0.2\|g_{k+1}\|^2$

is satisfied go to step 3,

else$k=k+1$ and go to step 4

3- Descent and the Sufficient Descent Conditions of the New Conjugate Gradient Algorithm (β_k^{New})

Theorem (3.1):- Assume that the sequence $\{x_k\}$ is generated by the form (1.2), where α_k is determined by the Wolfe line search (1.14) and (1.15) then the d_{k+1} given by (1.3) with modified CG-method in form (2.3) is a descent direction, i.e. $d_{k+1}^T g_{k+1} \le 0$ in both cases: exact and inexact line search.

Proof:

From (1.3) and (2.3), we have

$$d_{k+1} = -g_{k+1} + (\beta_k^{HS} - t\frac{\alpha_k \|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2})d_k \quad (3.1)$$

Multiply both sides by g_{k+1}^T, we get

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + (\beta_k^{HS} - t\frac{\alpha_k \|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2})g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{HS} g_{k+1}^T d_k - t\frac{\alpha_k \|d_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} \quad (3.2)$$

The proof is complete if the step length α_k is chosen by an exact line search which requires$d_k^T g_{k+1} = 0$. Now, if the step length α_k is chosen by an inexact line search which requires$d_k^T g_{k+1} \ne 0$. It is clearly the first two term of equation (3.2) is less than or equal to zero, and we know that $t, \alpha_k, \|d_k\|^2, (g_{k+1}^T d_k)^2$and

$(d_k^T y_k)^2$are positive we get to the third term of equation (3.2) is less than to zero.

So, we have $-t\frac{\alpha_k \|d_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} \le 0$

Finally, we have

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{HS} g_{k+1}^T d_k - t\frac{\alpha_k \|d_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} \le 0$$

Then the proof is complete.

Theorem (3.2): Suppose that the search direction d_k given by (1.3) and (2.3).We assume that the step length α_k satisfies strong Wolfe conditions (1.11) and (1.12). Then, the following result:

$$g_{k+1}^T d_{k+1} \le -c\|g_{k+1}\|^2$$

holds for any $k \ge 0$.

Proof

For the initial direction k=0, we have

$d_0 = -g_0 \Rightarrow d_0^T g_0 = -g_0^T g_0 \le -\|g_0\|^2$, which satisfied

Now, we suppose that $d_k^T g_k \le 0$,

$\forall\ i = 1,2,\ldots,k$, multiplying (1.3) by g_{k+1}^T, we get:

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \beta_k^{New} g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \left(\beta_k^{HS} - t\frac{\alpha_k \|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2}\right) g_{k+1}^T d_k$$

$$g_{k+1}^T d_{k+1} = -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k}{d_k^T y_k} g_{k+1}^T d_k - t\frac{\alpha_k \|d_k\|^2 (g_{k+1}^T d_k)^2}{(d_k^T y_k)^2} \quad (3.3)$$

It is clearly that t , α_k , $\|d_k\|^2$, $(g_{k+1}^T d_k)^2$ and $\left(d_k^T y_k\right)^2$ are positive, then we get to the third term of equation (3.3) is less than or equal to zero.

So, we have $\left(-t\frac{\alpha_k \|d_k\|^2 \left(g_{k+1}^T d_k\right)^2}{\left(d_k^T y_k\right)^2}\right) \leq 0$, then equation (3.3), we get

$$g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \frac{g_{k+1}^T y_k}{d_k^T y_k} g_{k+1}^T d_k \quad (3.4)$$

Since $g_{k+1}^T d_k \leq d_k^T y_k$,equation (3.4) becomes

$$g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + g_{k+1}^T y_k,$$

Now, we apply the inequality

$g_{k+1}^T y_k \leq \|g_{k+1}\| \|y_k\|$, hence

$$g_{k+1}^T d_{k+1} \leq -\|g_{k+1}\|^2 + \|g_{k+1}\| \|y_k\|,$$

$$\therefore\ g_{k+1}^T d_{k+1} \leq -\left(1 - \frac{\|y_k\|}{\|g_{k+1}\|}\right) \|g_{k+1}\|^2$$

Finally, we have

$$g_{k+1}^T d_{k+1} \leq -c\|g_{k+1}\|^2,$$

Where $c = 1 - \frac{\|y_k\|}{\|g_{k+1}\|} > 0$

Then the proof is complete.

4- The Global Convergence Analysis of the New Conjugate Gradient Algorithm (β_k^{New})

In order to establish the global convergence of new method, we need the following basic assumptions on the objective function.

Assumption (H).

i. The level set $S = \{x : x \in R^n, f(x) \leq f(x_1)\}$ is bounded, where x_1 is the starting point.
ii. In a neighborhood Ω of S, f is continuously differentiable and its gradient g is Lipschitz continuously, namely, there exists a constant $L > 0$ such that

$$\|g(x) - g(x_k)\| \leq L\|x - x_k\|, \forall x, x_k \in \Omega \quad (4.1)$$

Under these assumptions on f there exists a constant $\gamma \geq 0$,

such that $\|g(x)\| \leq \gamma, \quad \forall x \in S$.

Lemma (4.1). Suppose that the assumption (H) holds and consider any conjugate gradient (1.2) and (1.3), where is a descent direction d_k and α_k is obtained by the strong Wolfe line search. If

$\sum_{k\geq 1} \frac{1}{\|d_k\|^2} = \infty$, (4.2)

Then
$\lim_{k\to\infty} inf\|g_k\| = 0$. See (Dai and Yuan (1999)) (4.3)

If f is a uniformly convex function, there exists a constant $\vartheta > 0$ such that:

$\left(g(x) - g(y)\right)^T (x-y) \geq \vartheta\|x-y\|^2 \in \Omega$ (4.4)

We can rewrite (4.4) in the following manner:

$y_k^T v_k \geq \vartheta\|v_k\|^2$. (4.5)

Theorem (4.1): Suppose the assumption (H) holds and that f is a uniformly convex function. The new algorithm of the form (1.2), (1.3) and (2.3) where d_k satisfies the descent condition and$\alpha_k$is obtained by the strong Wolfe conditions (1.11) and (1.12) satisfies the global convergence.

(i.e.)$\lim_{k\to\infty} inf\|g_{k+1}\| = 0$

Proof:

From (1.3) and (2.3), we get

$d_{k+1} = -g_{k+1} + \beta_k^{New} d_k$, (4.6)

$\left|\beta_k^{New}\right| = \left|\frac{g_{k+1}^T y_k}{d_k^T y_k} - t\frac{\alpha_k\|d_k\|^2 g_{k+1}^T d_k}{(d_k^T y_k)^2}\right|$.

Since $g_{k+1}^T d_k \leq d_k^T y_k$,

$\left|\beta_k^{New}\right| \leq \left|\frac{g_{k+1}^T y_k}{d_k^T y_k}\right| + \left|t\frac{\alpha_k\|d_k\|^2}{d_k^T y_k}\right|$, (4.7)

From (4.5) it follows that

$\vartheta\|v_k\|^2 \leq y_k^T v_k$,

Implies that ,

$y_k^T d_k \geq \frac{\vartheta\|v_k\|^2}{\alpha_k}$,

Since$g_{k+1}^T y_k \leq \|g_{k+1}\|\|y_k\|$ and from Lipschitz Condition $\|y_k\| \leq L\|v_k\|$. Then

$\left|\beta_k^{New}\right| \leq \frac{\alpha_k L\|g_{k+1}\|}{\vartheta\|v_k\|} + t\frac{{\alpha_k}^2\|d_k\|^2}{\vartheta\|v_k\|^2}$,

Implies that

$\left|\beta_k^{New}\right| \leq \frac{\alpha_k L\gamma}{\vartheta\|v_k\|} + \frac{t}{\vartheta}$, (4.8)

Since $\|d_{k+1}\| \leq \|g_{k+1}\| + \left|\beta_k^{New}\right|\|d_k\|$, (4.9)

Then

$\|d_{k+1}\| \leq \gamma + (\frac{\alpha_k L\gamma}{\vartheta\|v_k\|} + \frac{t}{\vartheta})\|d_k\|$,(4.10)

$\|d_{k+1}\| \leq \gamma + (\frac{L\gamma}{\vartheta} + \frac{t}{\alpha_k\vartheta}\|v_k\|)$.

Since $\|v_k\| = \|x - x_k\|$,

$$D = \max\{\|x - x_k\|\}, \forall x, x_k \in R\}$$

Hence (4.10) becomes

$\|d_{k+1}\| \leq \gamma + (\frac{L\gamma}{\vartheta} + \frac{tD}{\alpha_k\vartheta}) = \varphi$.

$$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|^2} \geq \sum_{k\geq 1}\frac{1}{\varphi^2} = \sum_{k\geq 1} 1 = \infty$$

$\sum_{k\geq 1}\frac{1}{\|d_{k+1}\|^2} = \infty$ .By using lemma (1), we get

$$\lim_{k\to\infty} inf\|g_{k+1}\| = 0$$

5- Numerical results

This section is devoted to test the implementation of the new method. We compare the new conjugate gradient algorithm (New) and standard (H/S).The comparative tests involve well known nonlinear problems (see Appendix) with different function $4 \leq n \leq 5000$. All programs are written in FORTRAN 95 language and for all cases the stopping condition $\|g_{k+1}\| \leq 1 \times 10^{-5}$and restart using Powell condition $\left|g_k^T g_{k+1}\right| \geq 0.2\|g_{k+1}\|^2$ are used. The line search routine was a cubic interpolation which uses function and gradient values. The results given in table (1) specifically quote the number of iteration NOI and the number of function NOF. Experimental results in table (1) confirm that the new conjugate gradient algorithm (New) is superior to standard algorithm (H/S) with respect to the number of iterations NOI and the number of functions NOF.

Table (1) Comparing the Performance of the Two Algorithms of Standard (**H**/**S**) and (New)

Number of problem	N	Standard formula (HS)		New formula (New)	
		NOI	NOF	NOI	NOF
1	4	5	14	5	14
	100	5	14	5	14
	500	6	16	5	14
	1000	6	16	5	14
	5000	6	16	5	14
2	4	11	24	11	24
	100	49	99	49	99
	500	52	105	50	101
	1000	70	141	50	101
	5000	165	348	146	309
3	4	3	11	3	11
	100	14	81	11	57
	500	21	124	13	67
	1000	23	128	14	73
	5000	31	159	16	89
4	4	8	45	8	46
	100	49	185	47	166
	500	112	353	105	315
	1000	156	473	154	467
	5000	256	774	282	843
5	4	28	85	31	102
	100	33	114	32	104
	500	40	146	35	122
	1000	46	176	35	122
	5000	54	211	43	160
6	4	24	64	23	64
	100	29	79	29	79
	500	F	F	29	79
	1000	29	79	29	83
	5000	30	81	30	83
7	4	22	159	22	158
	100	22	159	22	158
	500	23	171	23	170
	1000	23	171	23	170
	5000	30	272	30	272
8	4	30	83	30	83
	100	30	83	30	85
	500	30	83	19	54
	1000	30	83	22	61
	5000	30	83	18	53

	4	38	108	31	92
	100	40	122	31	92
9	500	41	124	34	109
	1000	41	124	34	109
	5000	41	124	34	109
Total		**1890**	**6268**	**1703**	**5611**

Note: The fail result in standard CG is considered a twice value of new CG results.

Table (2) Comparing the Rate of Improvement between the New Algorithm (New) and the Standard Algorithm (H/S)

Tools	Standard algorithm (H/S)	New algorithm (New)
NOI	100%	90.1058%
NOF	100%	89.5182%

Table (2) shows the rate of improvement in the new algorithm (New) with the standard algorithm (H/S). The numerical results of the new algorithm are better than the standard algorithm. As we notice that (NOI), (NOF) of the standard algorithm are about 100%. That means the new algorithm has improvement as compared to standard algorithm with (9.8942%) in (NOI) and (10.4818%) in (NOF). In general, the new algorithm (New) has been improved by (10.188%) as compared to standard algorithm (H/S).

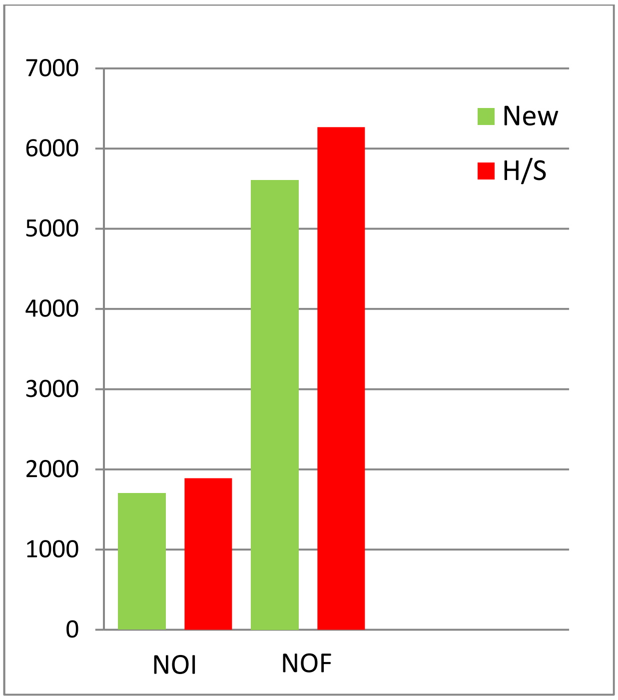

Figure (1): shows the comparison between new algorithm (New) and the standard algorithm (H/S) according to the total number of iterations (NOI) and the total number of functions (NOF).

6- Conclusion

In this paper, we have presented a new conjugate gradient method based on step size of Barzilai and Borwein, the descent, sufficient descent conditions and global convergence are proved and comparative numerical performances of well-known conjugate gradient algorithm (H/S) by using some standard test functions. Numerical results have shown that our new formula (β_k^{New})performs better than (H/S).

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Appendix

Test problems:

1-Generalized Edger Function:

$$f(x)=\sum_{i=1}^{n/2}(x_{2i-1}-2)^2+(x_{2i-1}-2)^2x_{2i}^2+(x_{2i}+1)^2$$

$$,\quad x_0=(1,0,\ldots,1,0)^T.$$

2-Wolfe function:

$$f(x)=\left(-x_1\left(3-\frac{x_1}{2}\right)+2x_2-1\right)^2+\sum_{i=1}^{n-1}\left(x_{i-1}-x_i\left(3-\frac{x_i}{2}+2x_{i+1}-1\right)\right)^2$$

$$+\left(x_{n-1}-x_n\left(3-\frac{x_n}{2}\right)-1\right)^2, x_0=(-1,\ldots,-1)^T.$$

3- Sum of Quadrics (SUM) Function:

$$f(x)=\sum_{i=1}^{n}(x_i-i)^4\quad,\quad x_0=(1,1,\ldots,1)^T.$$

4- Oren and Spedicato OSP Function:

$$f(x)=(\sum_{i=1}^{n}i(x_i)^2)^2 x_0=(1,\ldots,1)^T\ .$$

5-Miele Function:

$$f(x)=\sum_{i=1}^{n/4}(e^{x_{4i-3}}+10x_{4i-2})^2+100(x_{4i-2}+x_{4i-1})^6+(\tan(x_{4i-1}-x_{4i}))^4+(x_{4i-3})^8$$

$$+(x_{4i}-1)^2, x_0=(1,2,2,\ldots,1,2,2)^T.$$

6-Generalized non-diagonal Function:

$$f(x)=\sum_{i=2}^{n}100\left(x_1-x_i^2\right)^2+(1-x_i)^2, x_0=(-1,\ldots,-1)^T.$$

7-Generalized central Function:

$$f(x)=\sum_{i=1}^{n/4}(exp(x_{4i-3}+x_{4i-2})^4+100(x_{4i-2}-x_{4i-1})^6+arctan(x_{4i-1}-x_{4i})^4+x_{4i-3})\quad,$$

$$x_0=(1,2,2,2,\ldots,1,2,2,2)^T.$$

8-Generalized Rosen Brock Banana Function:

$$f(x) = \sum_{i=1}^{n/2} 100\left(x_{2i} - x_{2i-1}^2\right)^2 + (1 - x_{2i-1})^2$$

$$, \quad x_0 = (-1.2,1, \dots, -1.2,1)^T.$$

9-Powell Function:

$$f(x) = \sum_{i=1}^{n/4} ((x_{4i-3} - 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4),$$

$$x_0 = (3, -1,0,1, \dots, 3, -1,0,1)^T.$$

بو ختە:

دڤێ ڤەکۆلینێدا ، مە شێوازەکێ نوی یێ پێشنیارکری برێکا پەیسکێن هاوشێوە بۆچارەسەرکردنا ئاریشێن نموونەی یێن نەگرێدایپشت بەستن ب(step size of Barzilai and Borwein). پێشنیارامە یا نوی مەرجێن لاری ، لاریا کافی وسیفەتا نیزکیبونا گشتی بجهە ئینا. جیاوازیا ژمارەا دگەل ستاندەرێ پەیسکێن هاوشێوە بۆمە دیارکرد ئەڤ ئەلگوریزمە گەلك کارێکەرە پشت بەستن ب ژمارییێن دووبارە و ژمارە کردارێت نەخشان.

الخلاصة:

في هذا البحث ،تم اقتراح خوارزمية جديدة للتدرج المترافق لحل مسائل الامثلية الغير المقيدةبالاعتماد على طول الخطوة ل.(Barzilai and Borwein)خوارزمية التدرج المترافق التي اقترحنها تمتلك خاصية الانحدار وخاصية الانحدار الكافي وخاصية التقارب الشامل للخوارزمية المقترحة. النتائج العددية اثبتت بان الطريقة المقترحة اكثر كفاءة عند مقارنتها مع الطرق المشابهة لها في هذا المجال بالاعتماد على عدد التكرارات وعدد حسابات الدالة.