EDGE DEGREE WEIGHT OF SEQUENTIAL JOIN OF GRAPHS

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ABSTRACT

Let the weight w of an edge $e = uv = \{u,v\}$ of a graph G be defined by $w(e) = \deg(u) + \deg(v) - 2$ and the weight of G be defined by $w(G) = \sum_{e \in E(G)} w(e)$, where $E(G)$ is the edge set of *G*. In this paper the weights of joins, sequential joins, unions, intersections, and products (Cartesian and Tensor) of sets of graphs are obtained. This leads to a variety of open questions and new studies.

Keywords: Graph Theory, Edge degree weights

1. Introduction

Let G=(V(G), E(G)) be a simple graph of order p and size q . *Degree* of a vertex v in a graph G is the number of vertices incident with v and denoted by $\deg(v)$. Among many ways of assigning weights to the edges of a graph are those that are functions of the degrees of the vertices of a given edge $e = \{u,v\}$. Such weightings are of interest since the vertex degrees of a graph are intrinsic to the structure of the graph. One of the most natural of such weightings was studied by [Kattimani 2008] and [DuCasse and etal..2009] as:

$$w_0(e) = \deg(u) + \deg(v)$$

And extended to the weight of a graph G=(V(G), E(G)) as:

$$w_0(G) = \sum_{e \in E(G)} w_0(e)$$

Moreover, edge degree weight are generalizations by [DuCasse and Quintas 2011].

Here, we take one of the most natural types of vertex labeling, that of assigning the degree of a vertex as its label, and consider the induced labeling of the edge of a graph by assigning to an edge the sum of the degrees of the vertices that make up that edge minus two. This labeling possesses many properties and has non-trivial open problems associated with it.Our terminology are due to [Beineke and Wilson 1978] and [Buckely and Harary 1990].

Definition 1.1: The *weight* $w(e)$ of an edge $e = \{u,v\}$ is defined by: $w(e) = \deg(u) + \deg(v) - 2$ and the edge degree weight sum $w(G)$ of G relative to this edge weighting is $w(G) = \sum_{e \in E(G)} w(e)$.

The following Theorems are fundamental for the computation of $w(G)$.

Theorem 1.2: For any graph G,

$$w(G) = \sum_{v \in V(G)} (\deg(v))^2 - 2q(G).$$

Proof: By Definition 1.1, $w(e) = \deg(u) + \deg(v) - 2$ for edge $e = \{u,v\}$ in $E(G)$. Thus, each vertex v in $V(G)$ contributes the value $\deg(v) - 1$ to the weights of $\deg(v)$ edges and thus contributes $(\deg(v) - 1)\deg(v) = (\deg(v))^2 - \deg(v)$ to $w(G)$. Hence

$$w(G) = \sum_{v \in V(G)} ((\deg(v))^2 - \deg(v))$$

$$= \sum_{v \in V(G)} (\deg(v))^2 - \sum_{v \in V(G)} \deg(v)$$

Since, the sum of the degrees of the vertices of a graph is twice the number of degree, thus

$$w(G) = \sum_{v \in V(G)} (\deg(v))^2 - 2q(G).$$

Theorem 1.3: If G is regular graph of degree r, then $w(e) = 2r - 2$ for each edge $e = \{u,v\}$ in $E(G)$ and $w(G) = pr^2 - pr$.

Proof: By Definition 1.1, $w(e) = 2r - 2$. By Theorem 1.2,

$$w(G) = \sum_{v\in V(G)} (\deg(v))^2 - \sum_{v\in V(G)} \deg(v)$$

$$= \sum_{v\in V(G)} r^2 - \sum_{v\in V(G)} r = pr^2 - pr.$$

2. Remarks

The union $G = \bigcup_{i=1}^{k} G_i$ of k graphs is defined by

$$V(G) = \bigcup_{i=1}^{k} V(G_i) \text{ and } E(G) = \bigcup_{i=1}^{k} E(G_i).$$

It is trivial to observe that if the G_i are vertex disjoint, then the order, size, and weight of G are simply the sums of the orders, sizes, and weights, respectively of the G_i s. However, if the components are not vertex disjoint, then the order, size and weight of G cannot, in general, be expressed as functions of the orders, sizes and weights of the G_i s, without first determining the graph G.

Similarly, the order, size and weight of the intersection $H = \bigcap_{i=1}^{k} H_i$ of k graphs H_i, defined by

$$V(H) = \bigcap_{i=1}^{k} V(H_i) \neq \emptyset, \text{ and}$$

$$E(H) = \bigcap_{i=1}^{k} E(H_i)$$

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$$E(H) = \bigcap_{i=1}^{k} E(H_i)$$

Cannot, in general be expressed as functions of the orders, sizes and weights of the components without first determining the graph H.

3. Joins and Sequential Joins

The *join* $G_1 + G_2$ of two graphs G_1 and G_2 is a graph whose vertex set is $V(G_1+G_2) = V(G_1) \cup V(G_2)$ and $E(G_1+G_2) = E(G_1) \cup E(G_2) \cup \{uv : \forall u \in G_1 \text{ and } \forall v \in G_2\}$ with order $p_1 p_2$ and size $q_1 + q_2 + p_1 p_2$.

Theorem 3.1: Let G_1 and G_2 be two disjoint graphs of order p_1, p_2 and size q_1, q_2, respectively, then

$$w(G_1+G_2) = \begin{cases} w(G_1) + w(G_2) \\ +4(q_1 p_2 + q_2 p_1) + \\ p_1 (p_2)^2 + p_2 (p_1)^2 - 2p_1 p_2 \end{cases}$$

Proof: Let $\deg(u)$ and $\deg(v)$ be the degree of $u \in V(G_1)$ and $v \in V(G_2)$, respectively. Let $\deg^+(u)$ and $\deg^+(v)$, respectively, be the degree of u and v in the join $G_1 + G_2$. Then, $\deg^+(u) = \deg(u) + p_2, \deg^+(v) = \deg(v) + p_1$, and

$$w(G_1+G_2) = \begin{cases} \sum_{u\in V(G_1)} (\deg(u) + p_2)^2 \\ + \sum_{v\in V(G_2)} (\deg(v) + p_1)^2 \\ -2q(G_1+G_2) \end{cases}$$

$$= \begin{cases} \sum_{u\in V(G_1)} (\deg(u))^2 - 2q_1 \\ + \sum_{v\in V(G_2)} (\deg(v))^2 - 2q_2 \\ + \sum_{u\in V(G_1)} 2\deg(u) p_2 \\ + \sum_{v\in V(G_2)} 2\deg(v) p_1 + \sum_{u\in V(G_1)} (p_2)^2 \\ + \sum_{v\in V(G_2)} (p_1)^2 - 2p_1 p_2 \end{cases}$$

Noting that the sum of the degrees is equal to twice the size of the graph, it follows that

$$w(G_1+G_2)=\begin{cases} w(G_1)+w(G_2)+4q_1p_2 \\ +p_1(p_2)^2+4q_2p_1 \\ +p_2(p_1)^2-2p_1p_2 \end{cases}$$

$$=\begin{cases} w(G_1)+w(G_2)+ \\ 4(q_1p_2+q_2p_1)+ \\ p_1(p_2)^2+p_2(p_1)^2-2p_1p_2 \end{cases}$$

Corollary 3.2: Let G_1 and G_2 be two disjoint regular graphs of degree r_1 and r_2, respectively, then

$$w(G_1+G_2)=\begin{cases} p_1(r_1+p_2)^2+p_2(r_2+p_1)^2 \\ -(p_1r_1+p_2r_2+2p_1p_2) \end{cases}$$

Proof: By the above Theorem,

$$w(G_1+G_2)=\begin{cases} w(G_1)+w(G_2) \\ +4(q_1p_2+q_2p_1) \\ +p_1(p_2)^2+p_2(p_1)^2 \\ -2p_1p_2 \end{cases}$$

$$=\begin{cases} p_1r_1^2-p_1r_1+p_2r_2^2 \\ -p_2r_2+4(\frac{p_1r_1p_2}{2}+\frac{p_2r_2p_1}{2}) \\ +p_1(p_2)^2+p_2(p_1)^2-2p_1p_2 \end{cases}$$

$$=\begin{cases} p_1r_1^2-p_1r_1+p_2r_2^2 \\ -p_2r_2+2p_1r_1p_2+2p_2r_2p_1 \\ +p_1(p_2)^2+p_2(p_1)^2-2p_1p_2 \end{cases}$$

$$=\begin{cases} p_1(r_1^2+2r_1p_2+(p_2)^2) \\ +p_2(r_2^2+2r_2p_1+(p_1)^2) \\ -p_1r_1-p_2r_2-2p_1p_2 \end{cases}$$

$$=\begin{cases} p_1(r_1+p_2)^2+p_2(r_2+p_1)^2 \\ -(p_1r_1+p_2r_2+2p_1p_2) \end{cases}$$

Corollary 3.3: Let G_1 and G_2 be two disjoint regular graphs of the same degree r and the same order p, then

$$w(G_1+G_2)=2p(r+p)^2-2p(r+p)$$

Proof: Follows directly from the above Corollary.

The following Theorem concerns the sequential join of graphs. This graph operation is of interest in the study of distance problems in graphs. For concepts not explicitly defined here, see [Buckley and Harary 1990].

The *sequential join* $G_1+G_2+...+G_k$ of k graphs is the graph

$$(G_1+G_2)\cup(G_2+G_3)\cup...\cup(G_{k-1}+G_k)$$

where $+$ and $\cup$ are the *join* and *union* operations, respectively. See Fig.1.

Theorem 3.4: If G_i with $1\le i\le k$ are k disjoint graphs with orders p_i, sizes q_i, and $G=G_1+G_2+...+G_k$ is their sequential join, then

(1) The order of G is $\sum_{i=1}^{k} p_i$,

(2) The size of G is

$$q(G)=\sum_{i=1}^{k} q_i+\sum_{i=1}^{k-1} p_ip_{i+1}, \text{ and}$$

(3) The weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i)+4[q_1p_2+q_kp_{k-1} \\ +\sum_{i=2}^{k-1} q_i(p_{i-1}+p_{i+1})]+p_1(p_2)^2+ \\ (p_{k-1})^2p_k+\sum_{i=2}^{k-1} p_i((p_{i-1})^2+(p_{i+1})^2) \\ +2\sum_{i=2}^{k-1} p_{i-1}p_ip_{i+1}-2\sum_{i=1}^{k-1} p_ip_{i+1} \end{cases}$$

The graph shown in Fig. 1 can be used to illustrate Theorem 3.4. Note that, the weight of G obtained by Theorem 1.2 can verify the weight of G obtained by Theorem 3.4.

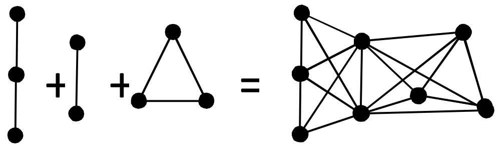

Figure 1: The sequential join graph
$G = P_3 + K_2 + K_3$.

Proof of Theorem 3.4:

(1) The order of G is obviously the sum of the orders of the G_i s.

(2) The size of G is the sum of the sizes of the G_i s plus the number of edges joining these graphs.

(3) Let x_{ij} denote the J^{th} vertex in G_i, $\deg(x_{ij})$ be the degree of x_{ij} in G_i and $\deg^+(x_{ij})$ be the degree of x_{ij} in G. Then,

$\deg^+(x_{ij}) = p_{i-1} + \deg(x_{ij}) + p_{i+1}$, where

$$p_0 = 0 = p_{k+1}.$$

From this one obtains, by Theorem 1.2,

$$w(G) = \begin{cases} \sum_{i=1}^{k}\sum_{j=1}^{p_i}(\deg^+(x_{ij}))^2 \\ -2\sum_{i=1}^{k} q_i - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

$$= \begin{cases} \sum_{j=1}^{p_i}(\deg(x_{1j}) + p_2)^2 \\ +\sum_{i=2}^{k-1}\sum_{j=1}^{p_i}(p_{i-1} + \deg(x_{ij}) + p_{i+1})^2 \\ -\sum_{j=1}^{p_k}(p_{k-1} + \deg(x_{kj}))^2 \\ -2q_1 - 2q_k - 2\sum_{i=2}^{k-1} q_i - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

$$= \begin{cases} \sum_{j=1}^{p_i}((\deg(x_{1j}))^2 + 2\deg(x_{1j})p_2 + (p_2)^2) \\ -\sum_{j=1}^{p_k}(p_{k-1} + \deg(x_{kj}))^2 \\ +\sum_{i=2}^{k-1}\sum_{j=1}^{p_i}[(\deg(x_{ij}))^2 \\ +2\deg(x_{ij})(p_{i-1} + p_{i+1}) + (p_{i-1})^2 \\ +(p_{i+1})^2 + 2p_{i-1}p_{i+1}] \\ -\sum_{j=1}^{p_k}(((\deg(x_{kj}))^2 \\ +2\deg(x_{kj})p_{k-1} + (p_{k-1})^2) - 2q_1 \\ -2q_k - 2\sum_{i=2}^{k-1} q_i - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

Noting that the sum of the degrees is equal to twice the size of a graph, it follows that

$$w(G) = \begin{cases} W(G_1) + 4q_1p_2 + p_1(p_2)^2 \\ +\sum_{i=2}^{k-1}[w(G_i) + 4q_i(p_{i-1} + p_{i+1}) \\ +p_i((p_{i-1})^2 + (p_{i+1})^2) \\ +2p_{i-1}p_i p_{i+1}] \\ +w(G_k) + 4q_k p_{k-1} + (p_{k-1})^2 p_k \\ -2\sum_{i=2}^{k-1} q_i - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

$$= \begin{cases} W(G_1) + w(G_k) + 4(q_1p_2 + q_k p_{k-1}) \\ +p_1(p_2)^2 + (p_{k-1})^2 p_k \\ +\sum_{i=2}^{k-1}[w(G_i) + 4q_i(p_{i-1} + p_{i+1}) \\ +p_i((p_{i-1})^2 + (p_{i+1})^2) + 2p_{i-1}p_i p_{i+1}] \\ -2\sum_{i=2}^{k-1} q_i - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

$$= \begin{cases} w(G_1) + w(G_k) + \sum_{i=2}^{k-1} w(G_i) \\ +4[q_1 p_2 + q_k p_{k-1} \\ +\sum_{i=2}^{k-1} q_i (p_{i-1} + p_{i+1})] + p_1 (p_2)^2 \\ +(p_{k-1})^2 p_k + \sum_{i=2}^{k-1} p_i ((p_{i-1})^2 + (p_{i+1})^2) \\ +2\sum_{i=2}^{k-1} p_{i-1} p_i p_{i+1} - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

The hypothesis of Theorem 3.4 contains two variables, namely, the order and the size of the k graphs. By letting the order be a constant p, or the size be a constant q, or the both the order and the size be constant, or by restricting the k graphs to be regular graphs, a variety of specialization of the general case covered by Theorem 3.4 can be obtained. These are given in the following table, where G is the sequential join $G_1 + G_2 + ... + G_k$:

Hypothesis 1: Put $p_i = p$.

Conclusion: **(1)** order of G is kp,

(2) Size of G is

$$\sum_{i=1}^{k} q_i + (k-1)p^2$$, and

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 4p(q_1 + q_k + 2\sum_{i=2}^{k-1} q_i) \\ +2(2k-3)p^3 - 2(k-1)p^2 \end{cases}$$

Hypothesis 2: Put $q_i = q$.

Conclusion: **(1)** order of G is $\sum_{i=1}^{k} p_i$,

(2) Size of G is $kq + \sum_{i=1}^{k-1} p_i p_{i+1}$, and

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 4q(p_1 + p_k + 2\sum_{i=2}^{k-1} p_i) \\ +p_1(p_2)^2 + (p_{k-1})^2 p_k \\ +\sum_{i=2}^{k-1} p_i ((p_{i-1})^2 + (p_{i+1})^2) \\ +2\sum_{i=2}^{k-1} p_{i-1} p_i p_{i+1} - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

Hypothesis 3: Put $p_i = p \ and \ q_i = q$.

Conclusion: **(1)** order of G is kp,

(2) Size of G is $kq + (k-1)p^2$, and

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 8(k-1)pq \\ +2(2k-3)p^3 - 2(k-1)p^2 \end{cases}$$

Hypothesis 4: For $1 \le i \le k$, G_i is $r_i - regular$ with order p_i, where $p_i r_i$ is even.

Conclusion: **(1)** order of G is $\sum_{i=1}^{k} p_i$,

(2) Size of G is $\frac{1}{2}\sum_{i=1}^{k} p_i r_i + \sum_{i=1}^{k-1} p_i p_{i+1}$, and

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 2[p_1 r_1 p_2 + p_k r_k p_{k-1} \\ +\sum_{i=2}^{k-1} p_i r_i (p_{i-1} + p_{i+1})] \\ +p_1(p_2)^2 + (p_{k-1})^2 p_k \\ +\sum_{i=2}^{k-1} p_i ((p_{i-1})^2 + (p_{i+1})^2) \\ +2\sum_{i=2}^{k-1} p_{i-1} p_i p_{i+1} - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

Hypothesis 5: For $1 \le i \le k$, G_i is $r_i - regular$ with fixed order p, where pr_i is even.

Conclusion: **(1)** order of G is kp,

(2) Size of G is

$$\frac{p}{2}\sum_{i=1}^{k} r_i + (k-1)p^2 \text{, and}$$

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 2p^2[r_1 + r_k \\ +2\sum_{i=2}^{k-1} r_i] + 2(2k-3)p^3 \\ -2(k-1)p^2 \end{cases}$$

Hypothesis 6: For $1 \le i \le k$, G_i is $r - regular$ with order p_i, where pr_i is even.

Conclusion: **(1)** order of G is $\sum_{i=1}^{k} p_i$,

(2) Size of G is

$$\frac{r}{2}\sum_{i=1}^{k} p_i + \sum_{i=1}^{k-1} p_i p_{i+1} \text{, and}$$

(3) Weight of G is

$$\begin{cases} \sum_{i=1}^{k} w(G_i) + 2r[p_1 p_2 \\ +p_k p_{k-1} + \sum_{i=2}^{k-1} p_i(p_{i-1} + p_{i+1})] \\ +p_1(p_2)^2 + (p_{k-1})^2 p_k \\ +\sum_{i=2}^{k-1} p_i((p_{i-1})^2 + (p_{i+1})^2) \\ +2\sum_{i=2}^{k-1} p_{i-1} p_i p_{i+1} - 2\sum_{i=1}^{k-1} p_i p_{i+1} \end{cases}$$

Hypothesis 7: For $1 \le i \le k$, G_i is $r - regular$ with fixed order p, where pr is even.

Conclusion: **(1)** order of G is kp,

(2) Size of G is

$$\frac{1}{2}kpr + (k-1)p^2 \text{, and}$$

(3) Weight of G is

$$\begin{cases} kpr^2 + 4(k-1)rp^2 \\ +2(2k-3)p^3 - 2(k-1)p^2 \end{cases}$$

4- Products

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be vertex disjoint non-trivial graphs. The **Cartesian product** $G_1 \times G_2$ of the two graphs G_1 and G_2 is the graph with vertex set $V(G_1 \times G_2) = V_1 \times V_2$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \times G_2$ if, and only if,

$[u_1 \equiv u_2 \; and \; v_1 \; adjacent \; with \; v_2 \; in \; G_2]$ or $[v_1 \equiv v_2 \; and \; u_1 \; adjacent \; with \; u_2 \; in \; G_1]$.

The following Theorem determine the weight of $G_1 \times G_2$.

Theorem 4.1: Let G_1 and G_2 be disjoint graphs of order p_1 and p_2, and size q_1 and q_2, respectively. Then

$$w(G_1 \times G_2) = p_2 w(G_1) + p_1 w(G_2) + 8q_1 q_2$$

Proof: Let $x \in V(G_1)$, $y \in V(G_2)$, and $(x, y) \in V(G_1 \times G_2)$. Then,

$$w(G_1 \times G_2) = \begin{cases} \sum_{(x,y)\in V(G_1\times G_2)} (\deg(x,y))^2 \\ -2q(G_1 \times G_2) \end{cases}$$

$$= \begin{cases} \sum_{(x,y)\in V(G_1\times G_2)} (\deg(x) + \deg(y))^2 \\ -2(p_1 q_2 + p_2 q_1) \end{cases}$$

$$= \begin{cases} \sum_{(x,y)\in V(G_1\times G_2)} [(\deg(x))^2 + 2\deg(x)\deg(y) \\ +(\deg(y))^2] - 2p_1 q_2 - 2p_2 q_1 \end{cases}$$

$$= \begin{cases} \sum_{(x,y)\in V(G_1\times G_2)} (\deg(x))^2 \\ +2\sum_{(x)\in V(G_1)} \deg(x) \sum_{(y)\in V(G_2)} \deg(y) \\ +\sum_{(x,y)\in V(G_1\times G_2)} (\deg(y))^2 \\ -2p_1q_2 - 2p_2q_1 \end{cases}$$

Since,

$w(G) + 2q(G) = \sum_{v\in V(G)} (\deg(v))^2$, thus

$$w(G_1 \times G_2) = \begin{cases} p_2(w(G_1) + 2q_1) \\ +2(2q_1)(2q_2) + p_1(w(G_2) \\ +2q_2) - 2p_1q_2 - 2p_2q_1 \end{cases}$$

$$= \begin{cases} p_2 w(G_1) + 2p_2q_1 + 8q_1q_2 + p_1 w(G_2) \\ +2p_1q_2 - 2p_1q_2 - 2p_2q_1 \end{cases}$$

$$= p_2 w(G_1) + p_1 w(G_2) + 8q_1q_2 .$$

Corollary 4.2: Let G_1 and G_2 be disjoint regular graphs of order p_1 and p_2, and degree r_1 and r_2, respectively. Then

$w(G_1 \times G_2) = p_1p_2(r_1 + r_2)^2$.

Proof: From **Theorem 4.1**,

$w(G_1 \times G_2) = p_2 w(G_1) + p_1 w(G_2) + 8q_1q_2$,

where q_1 and q_2 are the respectively sizes of G_1 and G_2. In this case, $w(G_1) = p_1 r_1^2$, $w(G_2) = p_2 r_2^2$, $q_1 = \frac{p_1r_1}{2}$, and $q_2 = \frac{p_2r_2}{2}$.

Thus,

$$w(G_1 \times G_2) = p_2 p_1 r_1^2 + p_1 p_2 r_2^2 + 8\frac{p_1r_1}{2}\frac{p_2r_2}{2}$$

Therefore, $w(G_1 \times G_2) = p_1p_2(r_1 + r_2)^2$.

Corollary 4.3: Let G_1 and G_2 be disjoint regular graphs of the same order p and degree r, then $w(G_1 \times G_2) = p^2(3r^2 + 2r)$.

The **tensor product** $G_1 \otimes G_2$ of the two graphs G_1 and G_2 is the graph with vertex set $V(G_1 \otimes G_2) = V_1 \times V_2$ and two vertices (u_1, v_1) and (u_2, v_2) are adjacent in $G_1 \otimes G_2$ if, and only if, $[u_1\ adjacent\ with\ u_2\ in\ G_1]$ and $[v_1\ adjacent\ with\ v_2\ in\ G_2]$.

The following Theorem determine the weight of $G_1 \otimes G_2$.

Theorem 4.4: Let G_1 and G_2 be disjoint graphs of order p_1 and p_2, and size q_1 and q_2, respectively. Then

$$w(G_1 \otimes G_2) = \begin{cases} w(G_1) w(G_2) + 2q_2 w(G_1) \\ +2q_1(G_2) \end{cases}.$$

Proof: Let $x \in V(G_1)$, $y \in V(G_2)$, and $(x, y) \in V(G_1 \times G_2)$. Then,

$$w(G_1 \otimes G_2) = \begin{cases} \sum_{(x,y)\in V(G_1\otimes G_2)} (\deg(x,y))^2 \\ -2q(G_1 \otimes G_2) \end{cases}$$

$$= \sum_{(x,y)\in V(G_1\otimes G_2)} (\deg(x)\deg(y))^2 - 4q_1q_2$$

$$= \sum_{(x)\in V(G_1)} \deg(x)^2 \sum_{(y)\in V(G_2)} \deg(y)^2 - 4q_1q_2$$

Since, $w(G) + 2q(G) = \sum_{v\in V(G)} (\deg(v))^2$, thus

$$w(G_1 \otimes G_2) = \begin{cases} (w(G_1) + 2q_1)(w(G_2) + 2q_2) \\ -4q_1q_2 \end{cases}$$

$$= w(G_1)w(G_2) + 2q_2 w(G_1) + 2q_1 w(G_2).$$

Corollary 4.5: Let G_1 and G_2 be disjoint regular graphs of order p_1 and p_2, and degree r_1 and r_2, respectively. Then

$w(G_1 \otimes G_2) = p_1p_2(r_1^2 r_2^2 + r_2 r_1^2 + r_1 r_2^2)$.

Proof: From Theorem 4.4,

$$w(G_1 \otimes G_2) = \begin{cases} w(G_1) w(G_2) + 2q_2 w(G_1) \\ +2q_1(G_2) \end{cases}$$

, where q_1 and q_2 are the respectively sizes of G_1 and G_2. In this case, $w(G_1) = p_1 r_1^2$, $w(G_2) = p_2 r_2^2$, $q_1 = \frac{p_1r_1}{2}$, and $q_2 = \frac{p_2r_2}{2}$.

Thus,

$$w(G_1 \otimes G_2) = \begin{cases} p_1 r_1^2 p_2 r_2^2 + 2\frac{p_2r_2}{2} p_1 r_1^2 \\ +2\frac{p_1r_1}{2} p_2 r_2^2 \end{cases}$$

Therefore,

$w(G_1 \otimes G_2) = p_1p_2(r_1^2 r_2^2 + r_2 r_1^2 + r_1 r_2^2)$.

Corollary 4.6: Let G_1 and G_2 be disjoint regular graphs of the same order p and degree r, then $w(G_1 \otimes G_2) = p^2(r^4 + 2r^3)$.

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وزن درجة الحافة للجمع التتابعي للبيانات

الملخص:

ليكن $W(e)$ وزن الحافة $e = uv = \{u,v\}$ في البيان G , عرفنا $w(e) = \deg(u) + \deg(v) - 2$ و $w(G)$ هو وزن البيان G معرفا" ب $w(e)$ $w(G) = \sum_{e \in E(G)} w(e)$ حيث أن $E(G)$ تمثل مجموعة الحافات للبيان G.

في هذا البحث تم ايجاد اوزان جمع البيانات وأوزان الجمع التتابعي للبيانات وأوزان اتحادها , تقاطعها , جداؤها الكارتيزي و جداؤها التنسري.

سەنگا پلا رەخێن كومكرنا ئێك ل ديف ئێك بو گرافا

كورتی

ئەگەر $W(e)$ سەنگا رەخا $e = uv = \{u,v\}$ بيت ل گرافێ G دا , , مە پێناسەكر $w(e) = \deg(u) + \deg(v) - 2$ و $w(G)$ سەنگا گرافێ G هاتە پێناسين كرن وەك $w(e)$ $w(G) = \sum_{e \in E(G)} w(e)$ كو $E(G)$ ئەكتەرێت كوما رەخێت گراف G.

دڤی ڤەكولينێدا هاتە دياركرن سەنگێن كومكرنا گرافا و سەنگا كومكرنا ئيچك ل ديف ئێك و سەنگا ئێككگرتنا وان و يەكتزبرينا وان . هەروەسا سەنگا لێكدانا كارتيزي و لێكدانا تنسوري.