MAXIMUM {SUPPLIES, DEMANDS} METHOD TO FIND THE INITIAL TRANSPORTATION PROBLEM

Dhia **A**bdul Sattar **K**adhem and **M**ardeen **S**hawkat **T**aher
Dept. of Mathematics, Faculty of Science, University of Duhok, Kurdistan Region – Iraq

(Accepted for publication: June 9, 2013)

ABSTRACT

In this paper, we have developed an additional method using the Maximum {Supplies, Demands} and combining both of them with the minimum cost to find an initial solution which is very close to the optimal or at most it is the optimum solution.

The transportation algorithm follows the exact steps of the simplex method. However, instead of using the regular simplex tableau, we take advantage of the special structure of the transportation model to organize the computation in a more convenient form

There are several methods for finding the initial basic feasible solution (BFS) of Transportation Problem (TP). But, there is no suitable answer to the question: Which method is the best one

1. Definition:

In general, a transportation problem is specified by the following information:

• A set of m supply points form which a good is shipped. Supply point i can supply at most s_i units.

• A set of n demand points to which the good is shipped. Demand point j must receive at least d_j units of the shipped good.

• Each unit produced at supply point i and shipped to demand point j incurs a variable cost of c_{ij} .

2. Formulating Transportation Problems:

Let x_{ij} = number of units shipped from supply point i to demand point j then the general LP representation of a transportation problem is

$min \sum_i \sum_j c_{ij} \quad x_{ij}$
$S.t.$

$\sum_j x_{ij} \; < \; s_i \; (i = 1,2,\ldots,m)$
$Supply\ constraints$
$\sum_i x_{ij} \; > \; d_j \; (j = 1,2,\ldots,n)$
$Demand\ constraints$
$x_{ij} \; > \; 0$

If a problem has the constraints given above and is a maximization problem, it is still a transportation problem. Finding BFS for Transportation Problems for a balanced transportation problem, the general LP representation may be written as:

$min \, \Sigma_i \, \Sigma_j \, c_{ij} \, x_{ij}$
$s.t.$

$\Sigma_j \, x_{ij} \; = \; s_i \; (i = 1,2,\ldots,m) \; Supply\ constraints$
$\Sigma_i \, x_{ij} \; = \; d_j \; (j = 1,2,\ldots,n) \; Demand\ constraints$
$x_{ij} \; > \; 0$

To find a BFS to a balanced transportation problem, we need to make the following important observations:

If a set of values for the $x_{ij}'s$ satisfies all but one of the constraints of a balanced transportation problem, the values for the $x_{ij}'s$ will automatically satisfy the other constraint.

This observation shows that when we solve a balanced transportation, we may omit from consideration any one of the problem's constraints and solve a LP having $m + n - 1$ constraints. We arbitrarily assume that the first supply constraint is omitted from consideration.

In trying to find a bfs to the remaining $m + n - 1$ constraints, you might think that any collection of $m + n - 1$ variables would yield a basic solution. But this is not the case:

If the $m + n - 1$ variables yield a basic solution, the cells corresponding to a set of $m + n - 1$ variables contain **no loop**.

There are several methods that can be used to find a BFS for a balanced transportation problem: [5]

1. Northwest Corner method
2. Minimum Cost method
3. Vogel's method

3. Algorithm of Max {Supplies, Demands} method:

Step 1: Choose the maximum supply from the sources i and maximum demand from the destinations j. Break ties arbitrarily

Step 2: Identify the row or column with the smallest shipping cost C_{ij} (call it x_{ij}) Then assign x_{ij} its largest possible value of supply or demand. Break ties arbitrarily

Step 3: Cross out the row or column with zero supply or demand to indicate that no further assignment can be made in that row or column. If both a row and a column net to zero simultaneously, cross out one only, and leave a zero supply (demand) in the uncrossed-out row (column)

Step 4:

- If exactly one row or column with zero supply or demand remains uncrossed out, stop.
- If one row (column) with positive supply (demand) remains uncrossed out, determine the basic variables in the row (column) by allocating the minimum cost. Stop.
- If all the uncrossed out rows and columns have (remaining) zero supply and demand, determine the zero basic variable by allocating the minimum cost. Stop. Otherwise, go to step 1

4. Example 1:

Three electric power fields with capacities of 70, 50, and 170 kWh supply electricity to five cities, the maximum demands at the five cities are estimated at 70,10,80,90, and 40 million kWh. The price per million kilo watt per hour (kw/h) at the five cities is given in the following table (1.a).

Table (1.a) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20	10	**70**
Field 2	12	25	30	10	60	**50**
Field 3	15	20	10	25	30	**170**
Demands	**70**	**10**	**80**	**90**	**40**	***290***

$Max\ \{supplies, demands\} = 170$

$Min(C_{ij}) = 10\ at\ the\ cell\ x_{33}\ or\ at\ the\ shipment\ from\ field\ 3\ to\ city\ 3$

$The\ required\ demand\ from\ the\ field\ 3\ to\ city\ 3 = 80$

After allocating the required supplies and demands we change the values of the supplies and the demands accordingly.

Table (1.b) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20	10	**70**
Field 2	12	25	30	10	60	**50**
Field 3	15	20	10 **80**	25	30	**90**
Demands	**70**	**10**	**0**	**90**	**40**	***210***

$Max\ \{supplies, demands\} = 90. \quad Break\ the\ tie\ arbiterily$

$Min(C_{ij}) = 10\ \ at\ the\ cell\ x_{24}\ or\ at\ the\ shipment\ from\ field\ 2\ to\ city\ 4$

$The\ required\ demand\ from\ the\ field\ 2\ to\ city\ 4 = 50$

Table (1.c) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20	10	**70**
Field 2	12	25	30	10 **50**	60	**0**
Field 3	15	20	10 **80**	25	30	**90**
Demands	**70**	**10**	**0**	**40**	**40**	***160***

$Max\,\{supplies, demands\} = 90.$
$Min(C_{ij}) = 15\ at\ the\ cell\ x_{31}\ or\ at\ the\ shipment\ from\ field\ 3\ to\ city\ 1$
$The\ required\ demand\ from\ the\ field\ 3\ to\ city\ 1 =\ 70$

Table (1.d) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20	10	**70**
Field 2	12	25	30	10 **50**	60	**0**
Field 3	15 **70**	20	10 **80**	25	30	**20**
Demands	**0**	**10**	**0**	**40**	**40**	***90***

$Max\,\{supplies, demands\} = 70.$
$Min(C_{ij}) = 10\ at\ the\ cell\ x_{15}\ or\ at\ the\ shipment\ from\ field\ 1\ to\ city\ 5$
$The\ required\ demand\ from\ the\ field\ 1\ to\ city\ 5 =\ 40$

Table (1.e) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20	10 **40**	**30**
Field 2	12	25	30	10 **50**	60	**0**
Field 3	15 **70**	20	10 **80**	25	30	**20**
Demands	**0**	**10**	**0**	**40**	**0**	***50***

$Max\,\{supplies, demands\} = 40.$
$Min(C_{ij}) = 20\ at\ the\ cell\ x_{14}\ or\ at\ the\ shipment\ from\ field\ 1\ to\ city\ 4$
$The\ required\ demand\ from\ the\ field\ 1\ to\ city\ 4 =\ 30$

Table (1.f) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20 **30**	10 **40**	**0**
Field 2	12	25	30	10 **50**	60	**0**
Field 3	15 **70**	20	10 **80**	25	30	**20**
Demands	**0**	**10**	**0**	**10**	**0**	***20***

Since we have left with one row which is field3, the supply will cover the demands for city 2 and city 4 accordingly, and this will complete the required demands.

Table (1.g) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20 **30**	10 **40**	**0**
Field 2	12	25	30	10 **50**	60	**0**
Field 3	15 **70**	20 **10**	10 **80**	25 **10**	30	**0**
Demands	**0**	**0**	**0**	**0**	**0**	***0***

Hence the initial basic feasible solution using the Maximum {Supplies, Demands} method is

Table (1.h) - Example 1

	City 1	City 2	City 3	City 4	City 5	Supplies
Field 1	10	30	40	20 **30**	10 **40**	**70**
Field 2	12	25	30	10 **50**	60	**50**
Field	15 **70**	20 **10**	10 **80**	25 **10**	30	**170**
Demands	**70**	**10**	**80**	**90**	**40**	***290***

Cost = 30*20+40*10+50*10+70*15+10*20+80*10+10*25 = 3,800

If we test the optimality of this initial basic feasible solution problem using the Stepping Stone Method [1], we will see that this solution is an optimal solution.

CONCLUSION

Our method does not depend only on the allocation as in the North West Corner Rule Method (NWCR), also it does not depend on the Least cost as in Least Cost Method (LCM) and Vogel's Approximation Method (VAM) but it takes into account the maximum supplies and the maximum demands and bound them together with the related minimum available cost from the sources and the demands respectively to reach an initial solution which is more closer to the optimal solution if it is not the optimal solution by itself as we have shown in our above example.

REFERENCES

Dantzig, G., Linear Programming and Extensions, Princeton University Press, Princeton, NJ, 1963.

Hillier F., and Lieberman G. Introduction to Operations Research. Holden-Day, 2001

Rardin, R.L., Optimization in Operations Research. Englewood Cliffs: Prentice Hall, 1998

Shri Deo Sharma and Dhia A. Kadhem, A new method for initial solution of transportation problem, Acta Ciencia Indica, Vol. XVM, No.1,11(1989)

Taha, H.A., Operations Research: An Introduction. Englewood Cliffs: Prentice Hall, Seven Edition, 2003

Winston W.L., Operations Research: Applications and Algorithms, Duxbury Press, Wadsworth Inc. 2004,

الملخص

في هذا البحث ، تم استحداث طريقة جديدة إضافية باستخدام أعلى {التجهيزات ، الطلبات} وربطهما مع الكلفة الأقل لإيجاد الحل الإبتدائي والذي سيكون قريبا جدا من الحل الامثل أو على الأغلب سيكون هو الحل الأمثل.

خوارزمية النقل تتبع نفس خطوات الطريقة السمبلكسية . ومع ذلك بدلا من استخدام جداول الطريقة السمبلكسية فاننا سنستفيد من الطرق الخاصة لحل مسائل النقل بالطرق المناسبة والملائمة.

توجد عدة طرق لايجاد الحل الابتدائي المنظور لمسألة النقل ولكن لا يوجد جواب مناسب للتساؤل حول أي طريقة من هذه الطرق هي الافضل.

كورتی

دڤێ ڤەكولینێ دا، رێكەكا دی یا نوی هاتە دیاركرن بەكارئینانەكا بلند { بەرهەڤی ، داخوازی } و گرێدانا وان دگەل كێمترین نرخ بۆ دیتنا شیكارا سەرەتایی ئەوا دێ نێزیكی شیكارا نموونەیی بت یان ژی هەما پتریا جارا ئەو بخۆ شیكارا نموونەیی یە.

خوارزمیا ڤەگوهاستنێ دویڤ هەمان پێنگاڤێن رێكا سیمپلێكس دچت، ودگەل هندێ دا شووینا بەكارئینانا خشتێن رێكا سیمپلێكس ئەم دێ مفای بێنین ژ رێكێن تایبەت بۆ كێشەیێن ڤەگۆهاستنێ ب رێكێن باش و ژ هەژی.

گەلەك رێك یێن هەین بۆ ب دەست ڤەئینانا شیكارا سەرەتایی یا چاڤەریێ بۆ كێشەیا ڤەگوهاستنێ بەلێ پا چ بەرسڤێن ژ هەژی نینن بۆ پرسیاركرنێ ل دور كا كیژ رێك ژڤان رێكا چێترە.