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A CORRECTION TO THE PAPER ``ON A−TOPOLOGICAL VECTOR SPACES''

Madhu Ram

Department of Mathematics, University of Jammu, Jk-180006, India - (madhuram0502@gmail.com)

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ABSTRACT:
Theorem 3.30 in the published paper [1] is not correct. A valid reasoning concerning this error is given.

KEYWORDS: α−open sets, α−topological vector spaces.

1. INTRODUCTION

In the paper [1], Theorem 3.30 is not correct. Although the proof of this theorem is not given in [1], the proof has some subtle stages where any mistake can happen. Also, it is not easy to refute [1, Theorem 3.30] by counterexamples because constructing examples of α−topological vector spaces are themselves a tough task. However, a valid refutation of [1, Theorem 3.30] follows from below argument:

Consider a set $U \in \mu_0$, the collection of all α−open sets containing 0 in an α−topological vector space $(X_{(F)}, \tau)$. Then there exist α−open sets D in the topological field F containing 0 and V in X containing 0 such that $D.V \subseteq U$. Here notice that the existence of a balanced α−open set in X that is contained in U follows from the existence of a balanced α−open set in F that is contained in D. More precisely, if F = R. Then [1, Theorem 3.30] holds only if D contains a set of the form $(-\epsilon, \epsilon)$ for some $\epsilon > 0$. To conclude the argument, it is enough to give an α−open set in R which does not contain any interval of the form $(-\epsilon, \epsilon)$. Let $D = (-1, 1) - \{1/n : n \in N$, the set of natural numbers$\}$. Then D is α−open set in R containing 0 which does not contain any set of the form $(-\epsilon, \epsilon)$.

Fortunately, [1, Theorem 3.30] does not affect other results and arguments in [1], although Theorem 3.30 in [1] was employed in [1, Theorem 3.36]. The proof of [1, Theorem 3.36] still follows by a weaker form of balancedness.

Lemma. Let $(X_{(F)}, \tau)$ be an α−topological vector space. Then for every $V \in \mu_0$, there exists a symmetric $U \in \mu_0$ such that $U + U \subseteq V$.

Proof. Let $V \in \mu_0$. By definition of α−topological vector spaces, there exist $U_1, U_2 \in \mu_0$ such that $U_1 + U_2 \subseteq V$. Set $U = U_1 \cap U_2 \cap (-U_1) \cap (-U_2)$. By [1, Theorem 3.5], $-U_1, -U_2 \in \mu_0$. In [2], it is proved that the finite intersection of α−open sets is α−open. Consequently, $U \in \mu_0$ satisfying $U = -U$ and $U + U \subseteq V$. This completes the proof.

Theorem. Let $(X_{(F)}, \tau)$ be an α−topological vector space. Then X is α−T2 if and only if for every $x \neq 0$, there exists $U \in \mu_0$ which does not contain 0.

Proof. Direct part. Already proved in [1, Theorem 3.36]. Conversely, let x and y be two distinct points in X. Then $x-y \neq 0$ and hence by assumption, there exists $U \in \mu 0$ which does not contain $x-y$. In light of above lemma, there exists a symmetric $V \in \mu_0$ such that $V + V \subseteq U$. Fix $U_1 = x + V$ and $U_2 = y + V$. By [1, Theorem 3.5], $U_1 \in \mu_x$ and $U_2 \in \mu_y$. Finally, if $a \in U_1 \cap U_2$, then $a = x + v_1$ and $a = y + v_2$ for some $v_1, v_2 \in V \Rightarrow x-y = -v_1 + v_2 \in -V + V = V + V \subseteq U$, the impossible. Hence the proof follows.

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