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SOFT ξ-OPEN SETS IN SOFT TOPOLOGICAL SPACES

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ABSTRACT:
The objective of studing the current paper is to introduced a new class of soft open sets in soft topological spaces called soft ξ-open sets. Then soft ξ-open sets are used to study some soft topological concepts. Furthermore, the concept of soft ξ-continuous and almost soft ξ-continuous functionS are defined by using the soft ξ-open sets. Some properties and Characterizations of such functions are given.

KEYWORDS: Soft topology, soft continuous, ξ-open set.

1 INTRODUCTION

(Molodtsov, 1999) invistigated the notion of soft set theory as a new mathematical instrument to transact with uncertainties and applied it successfully to several ternd such as smothness of functions, game theory, Operation research, theory of probability, etc.
(Shabir & Naz, 2011) launched the research of soft topological spaces and defined basic concepts in the subject of soft topological spaces. Then, (Hussain & Ahmad, 2011), (Ahmad & Hussain, 2012), (Aygünoğlu & Aygün, 2012), (Zorlutuna, Akdag, Min, & Atmaca, 2012) add up to many concepts across the properties of soft topological space. (Kharal & Ahmad, 2011) defined the notion of soft mappings on soft classes. Then (Aygünoğlu & Aygün, 2012) introduced soft continuity of soft mappings, (Nazmul & Samanta, 2012) studied the neighbouehood properties in a soft topological space. (Chen, 2013) invistigated in soft topological spaces the concept of soft semi-open sets and investigated some of its properties. (Yumak & Kaymakcı, 2013) are defined Soft β-open sets and continued to study weak forms of soft open sets in soft topological spaces. Then, (Akdag & Ozkan, 2014a, 2014b) defined soft-continuous functions, soft b-open(soft b-closed) sets and soft α-open (soft α-closed) sets respectively. Such sets and some others sets like them were introduced in ordinary topological spaces (see, semi-open sets by (Levine, 1963), pre-open sets by (El-Monsef & El-Deeb, 1982), α-open set by (Njåstad, 1965), ξ-open sets introduced by (Hasan, 2010). Ps-open by (B Khalaf Alias & Baravan, 2009), Ps-open by S_C-open (B Khalaf Alias & Zanyar, 2010) etc.

2 PRELEMINARIES

Definition.2.1 (Molodtsov, 1999) Assume U is an initial universe set, $P(\mathrm{U})$ is the set of all subsets of U, and E is a set of parameters. A pair (C,E) is called a soft set over U, where C is a map from E into $P(U)$. In what follows up, the familly of all soft sets of (C,E) over U be denoted by $SS(U,E)$.

Definition.2.2 (Molodtsov, 1999) Assume (C,E), $(C_1,E) \widetilde{\in} SS(U,E)$. The soft set (C,E) is said to be a soft subset of (C_1,E) symbolized by $(\mathrm{C},\mathrm{E}) \widetilde{\sqsubseteq} (\mathrm{C}_1,\mathrm{E})$, if $C(p) \widetilde{\sqsubseteq} C_1(p)$, for each $p \in E$. Also, it is said that the soft sets (C,E) and (C_1,E) are soft equal if $(\mathrm{C},\mathrm{E}) \widetilde{\sqsubseteq} (\mathrm{C}_1,\mathrm{E})$ and $(\mathrm{C}_1,\mathrm{E}) \widetilde{\sqsubseteq} (\mathrm{C},\mathrm{E})$.

Definition.2.3 (Molodtsov, 1999) Assume I is considered as an arbitrary index set $\{(C_i,E):(C_i,E) \widetilde{\in} SS(U,E): \forall i \in I\}$, then

1. The soft union of whole (C_i,E) is the soft set $(C,E) \widetilde{\in} SS(U,E)$, where $C: E \to P(U)$ is defined as: $C(e) = \widetilde{\sqcup} \{C_i(e): i \in I\}$, for each e belongs to E. Symbotically, it is written as $(C,E) = \widetilde{\sqcup} \{(C_i,E): i \in I\}$.
2. The soft intersection of whole (C_i,E) is about the soft set $(C,E) \widetilde{\in} SS(U,E)$, where $C: E \to P(U)$ is defined as: $C(e) = \widetilde{\sqcap} \{(C_i(e): i \in I\}$ for each e belongs to E. Symbotically, it is written as $(C,E) = \widetilde{\sqcap} \{(C_i,E): i \in I\}$.

Definition.2.4 (Zorlutuna et al., 2012) Assume $(C,E) \widetilde{\in} SS(U,E)$. The soft set $(C_1,E) \widetilde{\in} SS(U,E)$ is the soft complement of (C,E), where $C_1: E \to P(U)$ defined as: $C_1(e) = U \backslash C(e)$, for each e belongs to E. Symbolically, it is written as $(C_1,E) = (C,E)^c$. Obviously, $(C,E)^c = (C^c,E)$. For soft subsets $(C_2,E), (C_3,E) \widetilde{\in} SS(U,E)$, we have

i. $\left((C_2,E) \widetilde{\sqcup} (C_3,E)\right)^c = (C_2,E)^c \widetilde{\sqcap} (C_3,E)^c$;
ii. $((C_2,E) \widetilde{\sqcap} (C_3,E))^c = (C_2,E)^c \widetilde{\sqcup} (C_3,E)^c$.

Definition.2.5 (Molodtsov, 1999) The soft set $(C,E) \widetilde{\in} SS(U,E)$, where $C(e) = \emptyset$, for each e belongs to E is called the $E-null$ soft set of $SS(U,E)$ and denoted by 0_E. The soft set $(C,E) \widetilde{\in} SS(U,E)$, where $C(e) = U$, for each e belongs to E is called the $E-absolute$ soft set of $SS(U,E)$ and saymbolled by 1_E.

Definition.2.6 (Zorlutuna et al., 2012) The soft set $(C,E) \widetilde{\in} SS(U,E)$ is called a soft point in U, denoted by e_c, if for the element $e \in E, C(e) \neq 0_E$ and $C(e^c) = 0_E$ for all $e^c \in E \backslash \{e\}$. The set that contains whole soft points of U is denoted by $SP(U,E)$. The soft point e_c is said to be in (C_1,E), denoted by $e_F \widetilde{\in} (C_1,E)$, if for $e \in E$ and $C(e) \widetilde{\sqsubseteq} C_1(e)$.

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Definition.2.7 (Allam, Ismail, & Muhammed, 2017) A soft set $(C,E) \tilde{\in} SS(U,E)$ is said to be a soft element , denoted by e_x if $C(e)=\{x\}, C(e^c)=\emptyset$ for all $e^c \in E\backslash\{e\}$. We shall say that

1. $e_x \tilde{\in} (C_1,E)$ reads as e_x is belonging to (C_1,E) if $C(e) \tilde{\subseteq} C_1(e)$;
2. The soft elements e_x and e_y are said to be two distinct soft elements if $x \neq y$.

Definition.2.8 (Zorlutuna et al., 2012) Assume $SS(U,E)$ and $SS(Y,A)$ be two families of soft sets. Let $u: U \to Y$ and $p: E \to A$ be two maps. Then the mapping $f_{pu}: SS(U,E) \to SS(Y,A)$ is defined as

1. The image of $(C,E) \tilde{\in} SS(U,E)$ under f_{pu} is the soft set $f_{pu}(C,E) = (f_{pu}(C),A) = p(E)$ in $SS(Y,A)$ such that

$$f_{pu}(C)(y) = \begin{cases} \tilde{\bigcup}_{x \in p^{-1}(y) \cap E} \left(u\left(C(x)\right), \ p^{-1}(y) \cap E \neq \emptyset\right) \\ \phi, \quad otherwise \end{cases}$$

for all $y \in A$.

2. The invers image of $(C,A) \tilde{\in} SS(Y,A)$ under f_{pu} is the soft set $f_{pu}^{-1}(C,A) = (f_{pu}^{-1}(C_1), p^{-1}(A))$ in $SS(U,E)$ such that

$$f_{pu}^{-1}(C_1)(x) = \begin{cases} u^{-1}\left(C_1\left(p(x)\right)\right), & p(x) \in A \\ \emptyset, & otherwise \end{cases}$$

Proposition.2.9 (D. N. Georgiou & Megaritis, 2014) Let $(C,E),(C_1,E) \tilde{\in} SS(U,E)$ and $(C_2,A),(C_3,A) \tilde{\in} (Y,A)$. The statements in following are hold:

1. If $(C,E) \tilde{\subseteq} (C_1,E)$, then $f_{pu}(C,E) \tilde{\subseteq} f_{pu}(C_1,E)$,
2. If $(C_2,A) \tilde{\subseteq} (C_3,A)$, then $f_{pu}^{-1}(C_2,A) \tilde{\subseteq} f_{pu}^{-1}(C_3,A)$,
3. $(C,E) \tilde{\subseteq} f_{pu}^{-1}(f_{pu}(C,E))$,
4. $f_{pu}(f_{pu}^{-1}(C_2,A)) \tilde{\subseteq} (C_2,A)$,
5. $f_{pu}^{-1}((C_2,A)^c) = (f_{pu}^{-1}(C_2,A))^c$,
6. $f_{pu}\left((C,E) \tilde{\cup} (C_1,E)\right) = f_{pu}(C,E) \tilde{\cup} f_{pu}(C_1,E)$,
7. $f_{pu}\left((C,E) \tilde{\cap} (C_1,E)\right) \tilde{\subseteq} f_{pu}(C,E) \tilde{\cap} f_{pu}(C_1,E)$,
8. $f_{pu}^{-1}\left((C_2,A) \tilde{\cup} (C_3,A)\right) = f_{pu}^{-1}(C_2,A) \tilde{\cup} f_{pu}^{-1}(C_3,A)$,
9. $f_{pu}^{-1}\left((C_2,A) \tilde{\cap} (C_3,A)\right) = f_{pu}^{-1}(C_2,A) \tilde{\cap} f_{pu}^{-1}(C_3,A)$,

Definition.2.10 (Zorlutuna et al., 2012) Consider U as an initial universe set and let E be considered a set of parameters, and $\tilde{\tau} \tilde{\subseteq} SS(U,E)$. It is said that the family $\tilde{\tau}$ a soft topology on U if the axioms in following are hold:

1. $0_E, 1_E \tilde{\in} \tilde{\tau}$,
2. If $(G,E),(G_1,E) \tilde{\in} \tilde{\tau}$, then $(G,E) \tilde{\cap} (G_1,E) \tilde{\in} \tilde{\tau}$,
3. If $(G_i,E) \tilde{\in} \tilde{\tau}$ for every $i \in I$, then $\tilde{\cup}\{(G_i,E): i \in I\} \tilde{\in} \tilde{\tau}$.

The triple $(U,\tilde{\tau},E)$ is said a soft topological space. The soft sets of $\tilde{\tau}$ are called soft open sets in U. Also the soft set (H,E) is said to be soft closed in $\tilde{\tau}$ if its complement $(H,E)^c$ is soft open in $\tilde{\tau}$. The family of soft closed sets is symbolled by $\tilde{\tau}^c$. .

Definition.2.11 Let $(U,\tilde{\tau},E)$ be a soft topological space and $(C,E) \tilde{\in} SS(U,E)$,

1. **(Shabir & Naz, 2011)** The soft closure of (C,E) is the soft set $Cl_s(C,E) = \tilde{\cap}\{(H,E): (H,E) \tilde{\in} \tilde{\tau}^c, \ (C,E) \tilde{\subseteq} (H,E)\}$.
2. **(Zorlutuna et al., 2012)** The soft interior of (C,E) is the soft set $Int_s(C,E) = \tilde{\cup}\{(G,E): (G,E) \tilde{\in} \tilde{\tau}, \ (G,E) \tilde{\subseteq} (C,E)\}$.

Definition.2.12 (Zorlutuna et al., 2012) A soft set $(G,E) \tilde{\in} (U,\tilde{\tau},E)$ is said to be a soft neighborhood (briefly: nbd_s) of a $e_c \tilde{\in} SP(U,E)$ if there exists (G_1,E) wich is a soft open set such that $e_c \tilde{\in} (G_1,E) \tilde{\subseteq} (G,E)$. The system of soft neighborhoods of a e_c, denoted by $N_{\tilde{\tau}}(e_c)$.

Definition.2.13 (Çağman, Karataş, & Enginoglu, 2011) Consider the soft topological space $(U,\tilde{\tau},E)$ and let $(G,E) \tilde{\in} SS(U,E)$ and $e_c \tilde{\in} SP(U,E)$. If each soft neighborhood of e_c soft intersect to (G,E) in some soft points rather than e_c itself, then e_c is called soft limit point of (G,E). The set of all limit points of (G,E) is denoted by $D_s(G,E)$ and called soft drived set of (G,E).

Definition.2.14 (Aygünoğlu & Aygün, 2012) Assume the soft topological space $(U,\tilde{\tau},E)$.

1. The base for $\tilde{\tau}$ is a subcollection B of $\tilde{\tau}$, if any member of $\tilde{\tau}$ can be expressed as a soft union of members of B,
2. The subbase for $\tilde{\tau}$ is a subcollection S of $\tilde{\tau}$, if the family of all finite soft intersections of members of S forms a base for $\tilde{\tau}$.

Definition 2.15 (Zorlutuna et al., 2012) Consider $(U,\tilde{\tau},E)$ and $(Y,\tilde{\tau}^*,A)$ as soft topological spaces, $u: U \to Y$ and $p: E \to A$ be two mappings, and e_c belongs to $SP(U,E)$.

1. The map $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft continuous at $e_c \tilde{\in} SP(U,E)$ if for each $(G,A) \tilde{\in} N_{\tilde{\tau}^*}(f_{pu}(e_c))$, there exists $(G_1,E) \tilde{\in} N_{\tilde{\tau}}(e_c)$ such that $f_{pu}(G_1,E) \tilde{\subseteq} (G,A)$.
2. The map $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft continuous on $(U,\tilde{\tau},E)$ if f_{pu} is soft continuous at every e_c in $(U,\tilde{\tau},E)$.

Definition.2.16 (Yuksel, Tozlu, & Ergul, 2014) Assume the soft topological space $(U,\tilde{\tau},E)$ and $(G,E) \tilde{\in} (U,\tilde{\tau},E)$.

1. (G,E) is called a soft regular open set in $(U,\tilde{\tau},E)$,if $(G,E) = Int_s(Cl_s(G,E))$. The family of all soft regular open subset is denoted by $SRO(U,E)$.
2. (H,E) is called a soft regular closed set in $(U,\tilde{\tau},E)$, if $(H,E) = Cl_s(Int_s(H,E))$. The family of all soft regular closed subsets is denoted by $SRC(U,E)$.

Definition.2.17 (Ramadhan & Sayed, 2018) Assume that $(U,\tilde{\tau},E)$ is a soft topological space and $(C,E) \tilde{\in} (U,\tilde{\tau},E)$.

1. The soft set (C,E) is said to be soft δ-open set if for every $e_c \tilde{\in} (C,E)$, there exists a soft regular open set (G,E) such that $e_c \tilde{\in} (G,E) \tilde{\subseteq} (C,E)$, denoted by $(C,E) \tilde{\in} S\delta O(U,E)$ i.e. a soft set (C,E) is soft δ -open set if $(C,E) = \tilde{\cup}\{(C_\lambda,E)$:where (C_λ,E) is soft regular open set for each $\lambda\}$. The complement of the soft δ-open set (C,E) is called soft δ-closed set, denoted by $(C^c,E) \tilde{\in} S\delta C(U,E)$.
2. A point $e_c \tilde{\in} SP(U,E)$ is called soft δ- interior point of (C,E) if there exists a soft δ-open set

(G_1, E) such that $e_c \tilde{\in} (G_1, E) \tilde{\sqsubseteq} (C, E)$. The set of all δ-interior points of (C, E) is called the soft δ-interior of (C, E) and is denoted by $Int_s^{\delta}(C, E)$.

3. A soft point $e_c \tilde{\in} SP(U, E)$ is said to be a soft δ-cluster point of (C, E) if for every soft regular open (G, E) containing of e_c we have $(C, E) \tilde{\sqcap} (G, E) \neq 0_E$. The set of all soft δ-cluster points of (C, E) is called soft δ-closure of (C, E) denoted by $Cl_s^{\delta}(C, E)$.

Definition.2.18 (D. Georgiou, Megaritis, & Petropoulos, 2013) Assume the soft topological space $(U, \tilde{\tau}, E)$ and $(C, E) \tilde{\in} (U, \tilde{\tau}, E)$

1. A soft point $e_c \tilde{\in} SP(U, E)$ is said to be a soft θ-interior point of (C, E) if there exists a soft open set (G, E) of e_c such that $Cl_s(G, E) \tilde{\sqsubseteq} (C, E)$.
2. The soft θ-interior of (C, E) is denoted by $Int_s^{\theta}(C, E)$ and defined as the soft union of every soft open sets in $(U, \tilde{\tau}, E)$ whose soft closures are soft contained in (C, E).
3. (C, E) is said to be a soft θ-open set if $Int_s^{\theta}(C, E) = (C, E)$. The set of all soft θ-open sets over U denoted by $\tilde{\tau}_{\theta}$ or $S\theta O(U, E)$.

Definition.2.19 (Hussain, 2014) Assume that $(U, \tilde{\tau}, E)$ to is a soft topological space and (C, E) be any soft subset of $(U, \tilde{\tau}, E)$. Then (C, E) is said to be soft semi-open set if and only if there exists a soft open set $(G, E) \tilde{\in} (U, \tilde{\tau}, E)$ such that $(G, E) \tilde{\sqsubseteq} (C, E) \tilde{\sqsubseteq} Cl_s(G, E)$. The set of all soft *semi*-open sets is symbolized as $SSO(U, E)$. A soft set (H, E) is saide to be soft *semi*-closed set if its relative complement is soft semi-open set. The set of all *semi*-closed sets is written as $SSC(U, E)$. Equvalently if there exists a soft closed set (H, E) such that $Int_s(H, E) \tilde{\sqsubseteq} (C, E) \tilde{\sqsubseteq} (H, E)$. Note that every soft open set is soft *semi*-open set, and any soft closed set is soft *semi*-closed set.

Definition.2.20 (Chen, 2013) Suppose $(U, \tilde{\tau}, E)$ is a soft topological space and $(C, E) \tilde{\in} SS(U, E)$, then

1. The soft $semi$-interior of (C, E) is denoted by $Int_s^S(C, E)$ and defined as $Int_s^S(C, E) = \tilde{\sqcup}\{(G, E): (G, E)$ is soft $semi$-open and $(G, E) \tilde{\sqsubseteq} (C, E)\}$.
2. The soft $semi$-closure of (C, E) is denoted by $Cl_s^S(C, E)$ and define as $Cl_s^S(C, E) = \tilde{\sqcap}\{(H, E): (H, E)$ is soft semi-closed and $(C, E) \tilde{\sqsubseteq} (H, E)\}$.

Theorem.2.21 (Zorlutuna et al., 2012) Consider $(U, \tilde{\tau}, E)$ and $(Y, \tilde{\tau},^* A)$ as two soft topological spaces. Suppose the soft function $f_{pu}: SS(U, E) \to SS(Y, A)$ and $e_c \tilde{\in} (U, \tilde{\tau}, E)$. Then, the following statements are equivalent:

1. f_{pu} is soft continuous at e_c;
2. For each $(G, A) \tilde{\in} N_{\tilde{\tau}^*}(f_{pu}(e_c))$, there exists a $(G_1, E) \tilde{\in} N_{\tilde{\tau}}(e_c)$ such that $(G_1, E) \tilde{\sqsubseteq} f_{pu}^{-1}(G, A)$;
3. For each $(G, A) \tilde{\in} N_{\tilde{\tau}^*}(f_{pu}(e_c))$, $f_{pu}^{-1}(G, A) \tilde{\in} N_{\tilde{\tau}}(e_c)$.

Definition.2.22 (D. Georgiou et al., 2013) Assume that $(U, \tilde{\tau}, E)$ and $(Y, \tilde{\tau}^*, A)$ are soft topological spaces. Let $u: U \to Y$ and $p: E \to A$ be mappings. Let $e_c \tilde{\in} SP(U, E)$, then

1. f_{pu} is soft θ-continuous at e_c if for each $(G, A) \tilde{\in} N_{\tilde{\tau}^*}(f_{pu}(e_c))$, there exists $(G_1, E) \tilde{\in} N_{\tilde{\tau}}(e_c)$ such that $f_{pu}\left(Cl_s^{\theta}(G_1, E)\right) \tilde{\sqsubseteq} (G, A)$.
2. If f_{pu} is soft θ-continuous at every soft point in $(U, \tilde{\tau}, E)$, then f_{pu} is called soft θ-continuous on $(U, \tilde{\tau}, E)$.

Definition.2.23 (Ramadhan & Sayed, 2018) Assume that $(U, \tilde{\tau}, E)$ and $(Y, \tilde{\tau}^*, A)$ are two soft topological spaces. Let $u: U \to Y$ and $p: E \to A$ be mappings and $e_c \tilde{\in} SP(U, E)$. The map $f_{pu}: SS(U, E) \to SS(Y, A)$ is called soft δ-continuous at e_c if for every (G, A) soft neighborhood of $f_{pu}(e_c)$, there exists a (G_1, E) soft neighborhood e_c such that $f_{pu}(Int_s(Cl_s(G_1, E))) \tilde{\sqsubseteq} Int_s(Cl_s(G, A))$.

Definition.2.24 (Mahanta & Das, 2012) A soft maping $f_{pu}: SS(U, E) \to SS(Y, A)$ is called soft $semi$-continuous function if the inverse image of each soft open set in (Y, A) is soft $semi$-open set in (U, E).

3 SOFT ξ-OPEN SETS IN SOFT TOPOLOGICAL SPACES

Definition.3.1 The soft open set (G, E) belongs to $(U, \tilde{\tau}, E)$ is said to be soft ξ-open set if for each soft element $e_x \tilde{\in} (G, E)$, there exists a soft *semi*-closed set (H, E) satisfying $e_x \tilde{\in} (H, E) \tilde{\sqsubseteq} (G, E)$.
The set of all soft ξ-open sets in $(U, \tilde{\tau}, E)$ is symbolized as $S\xi O(U, E)$. The soft set (H, E) is said to be soft ξ-closed if its relative complement is soft ξ-open and the set of all soft ξ-closed is symbolized as $S\xi C(U, E)$
Equvalently, a soft open subset (G, E) is soft ξ-open if it can be written as $(G, E) = \tilde{\sqcup}_{\lambda}\{(H_{\lambda}, E)\}$, where (G, E) is a soft ξ-open and (H_{λ}, E), is a soft *semi*-closed set for each λ.
It is obviouse from the definition that any soft ξ-open subset over a soft topological space $(U, \tilde{\tau}, E)$ is soft open, but the convers is not true in general as it is shown in the following example.

Example.3.2 Let $U = \{h_1, h_2\}$ $E = \{e_1, e_2\}$, $\tilde{\tau} = \{0_E, 1_E, (G, E)\}$ where $(G, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$. The family of soft closed sets consists of $0_E, 1_E, (G, E)^c$, where $(G, E)^c = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$. The family of all soft semi-open sets over U consits of $0_E, 1_E, (G, E), (C_1, E), (C_2, E)$ where $(C_1, E) = \{(e_1, \{h_1\}), (e_2, U)\}$ and $(C_2, E) = \{(e_1, U), (e_2, \{h_1\})\}$. The familly of soft *semi*-closed sets over U is $0_E, 1_E, (G, E)^c$, $(C_1, E)^c$, $(C_2, E)^c$ where $(G, E)^c = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$, $(C_1, E)^c = \{(e_1, ,\{h_2\}), (e\_2, \phi)\}$, $(C_2, E)^c = \{(e_1, \phi), (e_2, \{h_2\})\}$. Then the soft open $(G, E) = \{(e_1, \{h_1\}), (e_2, \{h_1\})\}$ is not soft ξ-open since the soft element $\{(e_1, \{h_1\}), (e_2, \phi)\} \tilde{\in} U \tilde{\not\sqsubseteq} (G, E)$.

Proposition.3.3 For $(G, E) \tilde{\in} (U, \tilde{\tau}, E)$, if $(G, E) \tilde{\in} S\delta O(U, E)$, then $(G, E) \tilde{\in} S\xi O(U, E)$.

Proof. Suppose that (G, E) is soft δ-open. This implies that (G, E) is open and for any $e_x \tilde{\in} (G, E)$, there exists a soft regular open set (G_1, E) satisfying $e_x \tilde{\in} (G_1, E) \tilde{\sqsubseteq} (G, E)$. Since (G_1, E) is soft regular open, implies that $(G_1, E) = Int_s(Cl_s(G_1, E)) \tilde{\sqsubseteq} (G, E)$ .This means that $e_x \tilde{\in} Int_s(Cl_s(G_1, E)) \tilde{\sqsubseteq} (G, E)$. Since $(G_1, E) = Int_s(Cl_s(G_1, E))$, then $Int_s(Cl_s(G_1, E))$ is *semi*-closed, then (G, E) is soft ξ-open set.

The example that followes, showes that convers of Proposition 3.3 may not holds in general.

Example.3.4 Let $U = \{1,2,3,...\}$, and $E = \{e_1, e_2\}$. The family of soft open sets $\{(G_i, E)$: where $(G_i, E)^c$ is finite for $i = 1,2,3,...\}$ with $0_E, 1_E$ forms a soft topological space. Thus, $S\xi O(U,E) = \tilde{\tau}$ when $S\delta O(U,E) = 0_E, 1_E$.

Corollary.3.5 Any soft θ-open set is soft ξ-open.

Proof. Follows directly from their definition.

Remark.3.6 The convers is not true as it is obvious from (Example.3.4)

Corollary.3.7 In $(U, \tilde{\tau}, E)$, if the soft open sets and soft closed sets are the same, then $S\xi O(U,E) = \tilde{\tau}$.

Proof. Suppose that in $(U, \tilde{\tau}, E)$ if the soft open sets and soft closed sets are the same, then every soft semi-open subset is soft closed in $(U, \tilde{\tau}, E)$ and so each soft semi-closed subset is soft open in $(U, \tilde{\tau}, E)$.

The converse of (Corollary.3.7) may not true in general as the following example showes:

Example.3.8 $U = \{h_1, h_2\}$ and $E = \{e_1, e_2\}$ and $\tilde{\tau} = \{0_E, 1_E, (G_1, E), (G_2, E), (G_3, E), (G_4, E), (G_5, E), (G_6, E), (G_7, E)\}$, where
$(G_1, E) = \{(e_1, \phi), (e_2, \{h_1\})\}$,
$(G_2, E) = \{(e_1, \phi), (e_2, \{h_2\})\}$,
$(G_3, E) = \{(e_1, \phi), (e_2, U)\}$,
$(G_4, E) = \{(e_1, \{h_2\}), (e_2, \phi)\}$,
$(G_5, E) = \{(e_1, \{h_2\}), (e_2, \{h_1\})\}$,
$(G_6, E) = \{(e_1, \{h_2\}), (e_2, \{h_2\})\}$,
and $(G_7, E) = \{(e_1, \{h_2\}), (e_2, U)\}$ is a soft topologybe defined over U, then the family of soft semi-closed sets are $0_E, 1_E$,
$\{(e_1, U), (e_2, \emptyset)\}$, $\{(e_1, U), (e_2, \{h_1\})\}$, $\{(e_1, U), (e_2, \{h_2\})\}$
$\{(e_1,\{h_2\}),(e_2,\phi)\}$, $\{(e_1,\{h_2\}),(e_2,\{h_2\})\}$
$\{(e_1,\{h_1\}),(e_2,\phi)\}$, $\{(e_1,\{h_1\}),(e_2,\{h_1\})\}$,
$\{(e_1,\{h_1\}),(e_2,\{h_2\})\}$, $\{(e_1, \{h_1\}), (e_2, U)\}$,
$\{(e_1,\phi),(e_2,\{h_1\})\}$, $\{(e_1,\phi),(e_2,\{h_2\})\}$ and $\{(e_1, \emptyset), (e_2, U)\}$.
It is noticeable that $SO(U,E) = S\xi O(U,E)$ while the soft open sets and soft closed sets are not the same.

Proposition.3.9 For any soft topological space $(U, \tilde{\tau}, E)$, a soft element belongs to $S\xi O(U,E)$ if and only if it belongs to $SRO(U,E)$.

Proof. Necessity, Let $e_x \tilde{\in} S\xi O(U,E)$. Then for all $e_x \tilde{\in} e\{x\}$, there exists a soft semi-closed set (H, E) such that $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} e\{x\}$. $(H,E) = e_x$ is soft open, it means that
$Int_s(Cl_s(H,E)) \tilde{\sqsubseteq} (H,E)$ (1)
Since $(H,E) \tilde{\sqsubseteq} Cl_s(H,E)$, then
$Int_s(H,E) = (H,E) \tilde{\sqsubseteq} Int_s(Cl_s(H,E))$ (2) (1)
From (1) and (2), (H, E) is soft regular open.
Sufficintly, it is obvious.

Proposition.3.10 If $\{(G_\lambda, E): \lambda \in \Delta\}$ is a collection of soft $\xi - open$ sets, then $\tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$ is soft ξ-open.

Proof. If $\{(G_\lambda, E): \lambda \in \Delta\}$ is a collection of soft ξ-open sets in $(U, \tilde{\tau}, E)$. Since (G_λ, E) is soft ξ-open set for each λ, then (G_λ, E) is soft open and hence $\tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$ is a soft open set in $(U, \tilde{\tau}, E)$.
Let $e_x \tilde{\in} \tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$. Then there exists $\lambda \in \Delta$ such that $e_x \in (G_\lambda, E)$. Since (G_λ, E) is soft ξ-open for each λ, so there exists a soft semi-closed set (H, E) such that $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} (G_\lambda, E) \tilde{\sqsubseteq} \tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$ so $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} \tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$, therefore, $\tilde{\sqcup}\{(G_\lambda, E): \lambda \in \Delta\}$ is a soft ξ-open in $(U, \tilde{\tau}, E)$.

Proposition.3.11 If the collection $\{(G_i, E): i = 1,2,3, ..., n\}$ is soft $\xi - open$ set for each $i = 1,2,3, ..., n$, then $\tilde{\sqcap}\{(G_i, E): i = 1,2, ..., n\}$ is soft ξ-open set.

Proof. Suppose that $\{(G_i, E): i = 1,2,3, ..., n\}$ is soft ξ-open sets in $(U, \tilde{\tau}, E)$, then $\{(G_i, E): i = 1,2,3, ..., n\}$ is soft open, therefore $\tilde{\sqcap}\{(G_i, E): i = 1,2, ..., n\}$ is soft open in $(U, \tilde{\tau}, E)$. Suppose that $e_x \tilde{\in} \tilde{\sqcap}\{(G_i, E): i = 1,2, ..., n\}$, then $e_x \tilde{\in} (G_i, E)$, but (G_i, E) is soft ξ-open for each i , so there exists soft *semi*-closed (H_i, E) such that $e_x \tilde{\in} (H_i, E) \tilde{\sqsubseteq} (G_i, E)$. Therefore, $\tilde{\sqcap}\{(G_i, E)$ is a soft ξ-open set. Hence, the family of all soft ξ-open subsets of $(U, \tilde{\tau}, E)$ forms a soft topology on (U, E) denoted by $\tilde{\tau}_\xi$.

Theorem.3.12 Assume that $(S, \tilde{\tau}, E)$ is a soft regular open subspace of $(U, \tilde{\tau}, E)$. If $(G, E) \tilde{\in} S\xi O(S, E)$, then $(G, E) \tilde{\in} S\xi O(U, E)$.

Proof. Let $(G, E) \tilde{\in} S\xi O(S, E)$. Since (S, E) is soft open in (U, E) , then (G, E) is open in (U, E) , and for all $e_x \tilde{\in} (G, E)$, there exists a soft *semi*-closed set (H, E) in (S, E) such that $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} (G,E)$, but $(S, E) \tilde{\in} SSC(U, E)$, then $(H, E) \tilde{\in} SSC(U, E)$. Therefore, $(G, E) \tilde{\in} S\xi O(U, E)$.
Theorem.3.13 In $(U, \tilde{\tau}, E)$, consider $(S, \tilde{\tau}, E)$ is a soft subspace of $(U, \tilde{\tau}, E)$ and $(S, \tilde{\tau}, E) \tilde{\in} SSC(U, E)$. If $(G, E) \tilde{\in} S\xi O(U, E)$ and $(G, E) \tilde{\sqsubseteq} (S, \tilde{\tau}, E)$, then $(G, E) \tilde{\sqsubseteq} S\xi O(S, E)$.

Proof. Consider $(G, E) \tilde{\in} S\xi O(U, E)$, and $(G, E) \tilde{\sqsubseteq} (S, \tilde{\tau}, E)$, then (G, E) is soft open in (S, E) and for all $e_x \in (G, E)$, there exists $(G_1, E) \tilde{\in} SSO(U, E)$ satisfying $e_x \tilde{\in} (G_1, E) \tilde{\sqsubseteq} (G, E)$, but $(S, \tilde{\tau}, E) \tilde{\in} SSC(U, E)$, then by separation axioms of soft sets, we get $(G_1, E) \tilde{\in} SSC(Y, E)$. Therefore $(G, E) \tilde{\sqsubseteq} S\xi O(Y, E)$.

Theorem.3.14 If $(S, \tilde{\tau}, E)$ is a soft open subspace of a soft space $(U, \tilde{\tau}, E)$ and $(G, E) \tilde{\in} S\xi O(U, E)$, then $(G, E) \tilde{\sqcap} (S, E) \tilde{\in} S\xi O(S, E)$.

Proof. Let $(G, E) \tilde{\in} S\xi O(U, E)$. Then (G, E) is a soft open set in (U, E) and $(G, E) = \{\tilde{\sqcup}(H_\lambda, E): (H_\lambda, E) \tilde{\in} SSC(U, E)$ for each $\lambda\}$, then by Definition 3.2. page 289**(Chen, 2013)**, $\tilde{\sqcup}\{(H_\lambda, E) \tilde{\sqcap} (S, E)\} \tilde{\in} SSC(S, E)$ for each $\lambda\}$. Also $(G, E) \tilde{\sqcap} (S, E)$ is soft open in (S, E) and $\tilde{\sqcup}\{(H_\lambda, E) \tilde{\sqcap} (S, E)\} \tilde{\sqsubseteq} (G, E) \tilde{\sqcap} (S, E)\}$. This implies that $(G, E) \tilde{\sqcap} (S, E) \tilde{\in} S\xi O(S, E)$.

4 SOFT ξ-OPERATORS ON SETS

Definition.4.1 A soft element $e_x \tilde{\in} (U, \tilde{\tau}, E)$ is said to be a soft ξ-interior point of (C, E) if there exists a soft ξ-open set (G, E) containing e_x such that $(G, E) \tilde{\sqsubseteq} (C, E)$. The set contains all soft ξ-interior points of (C, E) is symbolized as $Int_s^\xi(C, E)$.

Proposition.4.2 Assume that (C, E) is any soft subset of $(U, \tilde{\tau}, E)$. If the soft element $e_x \tilde{\in} Int_s^\xi (C, E)$, then there exists a soft semi-closed (H, E) of $(U, \tilde{\tau}, E)$ containing

e_x and $(H,E) \tilde{\sqsubseteq} (C,E)$.

Proof. It followes from Definition of soft ξ-interior and soft ξ-open sets.

Proposition.4.3 Suppose $(U,\tilde{\tau},E)$ is a soft topological space. For every soft subsets (C,E) and (C_1,E) of $(U,\tilde{\tau},E)$ the following statements hold,

1. $Int_s^{\xi}(0_E) = 0_E$ and $Int_s^{\xi}(1_E) = 1_E$,
2. The soft ξ-interior of (C,E) is the soft union of all soft ξ-open sets contained in (C,E),
3. $Int_s^{\xi}(C,E)$ is a soft ξ-open set in $(U,\tilde{\tau},E)$ contained in (C,E),
4. $Int_s^{\xi}(C,E)$ is the largest soft ξ-open set in $(U,\tilde{\tau},E)$ contained in (C,E),
5. (C,E) is a soft ξ-open if and only if $(C,E) = Int_s^{\xi}(C,E)$,
6. $Int_s^{\xi}(Int_s^{\xi}(C,E)) = Int_s^{\xi}(C,E)$,
7. $Int_s^{\xi}(C,E) \tilde{\sqsubseteq} (C,E)$,
8. If $(C,E) \tilde{\sqsubseteq} (C_1,E)$, then $Int_s^{\xi}(C,E) \tilde{\sqsubseteq} Int_s^{\xi}(C_1,E)$,
9. $Int_s^{\xi}(C,E) \tilde{\sqcup} Int_s^{\xi}(C_1,E) \tilde{\sqsubseteq} Int_s^{\xi}((C,E) \tilde{\sqcup} (C_1,E)$,
10. $Int_s^{\xi}(C,E) \tilde{\sqcap} Int_s^{\xi}(C_1,E) = Int_s^{\xi}((C,E) \sqcap (C_1,E))$.

Proof. 1) It is obviouse.
2) For all $e_{x_i} \tilde{\in} Int_s^{\xi}(C,E)$ implies that there exist soft $\xi - open$ sets (G_i,E) such that $e_{xi} \tilde{\in} (G_i,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$. $\tilde{\sqcup}\, e_{xi} \tilde{\sqsubseteq} \tilde{\sqcup}\, (G_i,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$
$\Rightarrow Int_s^{\xi}(C,E) \tilde{\sqsubseteq} \tilde{\sqcup}\, (G_i,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$
$\Rightarrow Int_s^{\xi}(C,E) = \tilde{\sqcup}\, (G_i,E)$
and (G_i,E) is soft ξ-open for each i.
3) By (2) $Int_s^{\xi}(C,E) = \tilde{\sqcup}\, \{(G_i,E): (G_i,E)$ is soft ξ-open for each i, $(G_i,E) \tilde{\sqsubseteq} (C,E)\}$. Thus $Int_s^{\xi}(C,E)$ is a soft ξ-open set which is a soft subset of (C,E). Now, Suppose that (G,E) is soft ξ -open and $e_x \tilde{\in} (G,E)$ such that $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C,E)$. Consequently e_x is a soft ξ-interior point of (C,E). Thus $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$. This shows that each soft ξ-open soft subset in (C,E) is soft contained in $Int_s^{\xi}(C,E)$. Hence, $Int_s^{\xi}(C,E)$ is the biggest soft ξ-open set enclosed in (C,E).
4,5,6,7,8 and 9 are Obviouse.

10) Since $(C,E) \tilde{\sqcap} (C_1,E) \tilde{\sqsubseteq} (C,E)$. Then by (8) $Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E)) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$ and $(C,E) \tilde{\sqcap} (C_1,E) \tilde{\sqsubseteq} (C_1,E)$, then $Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E)) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$ We get $Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E)) \tilde{\sqsubseteq} Int_s^{\xi}(C,E) \tilde{\sqcap} Int_s^{\xi}(C_1,E)$.
Conversly, let $e_x \tilde{\in} Int_s^{\xi}(C,E) \tilde{\sqcap} Int_s^{\xi}(C_1,E)$. So that $e_x \tilde{\in} Int_s^{\xi}(C,E)$ and $e_x \tilde{\in} Int_s^{\xi}(C_1,E)$, there exists $(G_1,E),(G_2,E) \tilde{\in} S\xi O(U,E)$ containing e_x such that $(G_1,E) \tilde{\sqsubseteq} (C,E)$, and $(G_2,E) \tilde{\sqsubseteq} (C_2,E)$. Put $(G_1,E) \tilde{\sqcap} (G_2,E) = (G_3,E)$ which is soft ξ-open set of (U,E), and $e_x \tilde{\in} (G_3,E) \tilde{\sqsubseteq} (C,E) \tilde{\sqcap} (C_1,E)$. Then $e_x \tilde{\in} Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E))$. We get $Int_s^{\xi}(C,E) \tilde{\sqcap} Int_s^{\xi}(C_1,E) \tilde{\sqsubseteq} Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E))$. Therefore, $Int_s^{\xi}((C,E) \tilde{\sqcap} (C_1,E)) = Int_s^{\xi}(C,E) \tilde{\sqcap} Int_s^{\xi}(C_1,E)$.

Generally $Int_s^{\xi}(C,E) \tilde{\sqcup} Int_s^{\xi}(C_1,E) \neq Int_s^{\xi}((C,E) \tilde{\sqcup} (C_1,E))$ as is displayed in the following example

Example.4.4 Consider $U = \{a,b,c,d\}$ and $E = = \{e_1\}$ with the soft topology $\tilde{\tau} = \{0_E, 1_E, \{(e_1, \{b,c\})\}, \{(e_1,\{d\})\}, \{(e_1,\{b,c,d\})\}, \{(e_1,\{a,b,c\})\}\}$.
$SC(U,E) = \{0_E, 1_E, \{(e_1,\{a,b,c\})\}, \{(e_1,\{a,d\})\} \{(e_1,\{a\})\}, \{(e_1,\{d\})\}\} = SSC(U,E)$.
$S\xi O(U,E) = \{0_E, 1_E, \{(e_1,\{a,b,c\})\}, \{(e_1,\{d\})\}\} = S\xi C(U,E)$.
Suppose that $(C,E) = \{(e_1,\{a,b\})\}$ and $(C_1,E) = \{(e_1,\{a,c\})\}$, then $(C,E) \tilde{\sqcup} (C_1,E) = \{(e_1,\{a,b,c\})\}$.
Now
$Int_s^{\xi}(C,E) = 0_E$ and $Int_s^{\xi}(C_1,E) = 0_E$, then $Int_s^{\xi}(C,E) \tilde{\sqcup} Int_s^{\xi}(C_1,E) = 0_E$ But $Int_s^{\xi}((C,E) \tilde{\sqcup} (C_1,E)) = Int_s^{\xi}(\{(e_1,\{a,b,c\})\}) = \{(e_1,\{a,b,c\})\}$.Hence, any soft ξ-open set is soft open and the soft ξ-interior point of a soft subset (C,E) of $(U,\tilde{\tau},E)$ is a soft interior point of (C,E), thus, $Int_s^{\xi}(C,E) \tilde{\sqsubseteq} Int_s(C,E)$.
In general, $Int_s^{\xi}(C,E) \neq Int_s(C,E)$ as displayed in the (Example.4.5)

Example.4.5 Consider $U = \{h_1,h_2\}$ and $E = \{e_1, e_2\}$ with the soft topology $(U,\tilde{\tau},E)$ that defined in (Example.3.2). If $(C,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}$, then $Int_s^{\xi}(C,E) = 0_E$, but $Int_s(C,E) = \{(e_1,\{h_1\}), (e_2,\{h_1\})\}$. Hence, $Int_s^{\xi}(C,E) \neq Int_s(C,E)$.

Definition.4.6 The soft ξ-closure of soft subset (C,E) is symbolized by $Cl_s^{\xi}(C,E)$and is soft intersection of any soft ξ-closed subsets containing (C,E).

Corollary.4.7 Assume that (C,E) is a soft subset of $(U,\tilde{\tau},E)$. A soft element $e_x \tilde{\in} SS(U,E)$ is in the soft ξ-closure of (C,E) if and only if $(C,E) \tilde{\sqcap} (G,E) \neq 0_E$ for each soft ξ-open set (G,E) containing e_x.

Proof. Followes directly from Definition.

Proposition.4.8 Assume that (C,E) is any soft subset of $(U,\tilde{\tau},E)$. If $(C,E) \tilde{\sqcap} (H,E) \neq 0_E$ for any soft $semi$-closed set (H,E) such that $e_x \tilde{\in} (H,E)$, then $e_x \tilde{\in} Cl_s^{\xi}(C,E)$.

Proof. Suppose that $(C,E) \tilde{\sqcap} (H,E) \neq 0_E$ for each soft $semi$-closed set (H,E) such that $e_x \tilde{\in} (H,E)$. And let $e_x \tilde{\notin} Cl_s^{\xi}(C,E)$. Then by Corollary 4.7, there exists a soft ξ-open (G,E) such that $e_x \tilde{\in} (G,E)$ and $(G,E) \tilde{\sqcap} (C,E) \neq 0_E$, and for each $e_x \tilde{\in} (G,E)$, there exists a soft $semi$-closed set (H_1,E) such that $e_x \tilde{\in} (H_1,E) \tilde{\sqsubseteq} (G,E)$, but $(H_1,E) \tilde{\sqsubseteq} (G,E) \tilde{\sqcap} (C,E)$
$= 0_E$ which is contradiction. Then $e_x \tilde{\in} Cl_s^{\xi}(C,E)$.
The converse of the above theorem generally need not be true as shown by the following example.
Example.4.9 Consider $U = \{a,b,c\}$ and $E = \{e_1\}$**.** Let the soft topology $\tilde{\tau}$ be defined on (U,E) as $\tilde{\tau} = 0_E, 1_E, \{(e_1,\{a\})\}, \{(e_1,\{b\})\}, \{(e_1,\{a,b\})\}$.
The $SSO(U,E) = \{0_E, 1_E, \{(e_1,\{a\})\}, \{(e_1,\{b\})\}, \{(e_1,\{a,b\})\}, \{(e_1,\{a,c\})\}, \{(e_1,\{b,c\})\}\}$.
$SSC(U,E) = \{(e_1,\{a\})\}, \{(e_1,\{b\})\}, \{(e_1,\{c\})\}, \{(e_1,\{a,c\})\}, \{(e_1,\{b,c\})\}\}$.
$S\xi O(U,E) = \{0_E, 1_E, \{(e_1,\{a\})\}, \{(e_1,\{b\})\}, \{(e_1,\{a, b\})\}$.
$S\xi C(U,E) = \{0_E, 1_E, \{(e_1,\{b,c\})\}, \{(e_1,\{a,c\})\}, \{(e_1,\{c\})\}\}$.

If we take $(C,E)=\{(e_1,\{a,b\})\}$, then $Cl_s^{\xi}(C,E)=1_E$. Then the soft element $\{(e_1,\{a\})\}$ is belong to the soft $semi$ -closed $\{(e_1,\{a\})\}$, such that $(C,E)\tilde{\sqcap}\{(e_1,$ $\{a\})\}\backslash(e_1,\{a\})\}=0_E$.

Theorem.4.10 For any soft subsets (C,E) and (C_1,E) of $(U,\tilde{\tau},E)$, the statements in following hold:

1. $Cl_s^{\xi}(0_E)=0_E$ and $Cl_s^{\xi}(1_E)=1_E$,
2. $Cl_s^{\xi}(C,E)$ is a soft ξ-closed set in $(U,\tilde{\tau},E)$ containing (C,E),
3. $Cl_s^{\xi}(C,E)$ is the smallest soft ξ-closed in $(U,\tilde{\tau},E)$ containing (C,E),
4. (C,E) is soft ξ-closed set if and only if $(C,E)=Cl_s^{\xi}(C,E)$,
5. $(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)$,
6. If $(C,E)\tilde{\sqsubseteq}(C_1,E)$, then $Cl_s^{\xi}(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C_1,E)$,
7. $Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)=Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))$,
8. $Cl_s^{\xi}((C,E)\tilde{\sqcap}(C_1,E))\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)\tilde{\sqcap}Cl_s^{\xi}(C_1,E)$.

Proof. As 1, 2, 3, 4 and 5 are easy, we prove(6)
(6) Choice $e_x\tilde{\in}Cl_s^{\xi}(C,E)$ and $(C,E)\tilde{\sqsubseteq}(C_1,E)$. Then there exists a soft $semi$-closed set (H,E) containing e_x we have $(C_1,E)\tilde{\sqcap}(H,E)\neq 0_E$. Therefore $(C,E)\tilde{\sqcap}(H,E)\neq 0_E$. Then $e_x\tilde{\in}Cl_s^{\xi}(C_1,E)$. Thus $Cl_s^{\xi}(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C_1,E)$.
(7) Since $(C,E)\tilde{\sqsubseteq}(C,E)\tilde{\sqcup}(C_1,E)$ and $(C_1,E)\tilde{\sqsubseteq}(C,E)\tilde{\sqcup}(C_1,E)$, so by part (4), $Cl_s^{\xi}(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))$ and $Cl_s^{\xi}(C_1,E)\tilde{\sqsubseteq}Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))$. Thus $Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)\tilde{\sqsubseteq}Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))$.
Conversly, suppose that $(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)$ and $(C_1,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C_1,E)$ so $(C,E)\tilde{\sqcup}(C_1,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)$, $Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)$ is a soft ξ-closed set over $(U,\tilde{\tau},E)$ being the union of two soft ξ-closed sets. Then $Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)$. Thus, $Cl_s^{\xi}((C,E)\tilde{\sqcup}(C_1,E))=$ $Cl_s^{\xi}(C,E)\tilde{\sqcup}Cl_s^{\xi}(C_1,E)$.
(8) It follows from part (4).
In general $Cl_s^{\xi}((C,E)\tilde{\sqcap}(C_1,E))\neq Cl_s^{\xi}(C,E)$ $\tilde{\sqcap}Cl_s^{\xi}(C_1,E)$ as is presented in the next example.

Example.4.11 Consider the soft topology $(U,\tilde{\tau},E)$ as defined in (Example.4.4). If we take $(C,E)=\{(e_1,\{a,d\})\}$ and $(C_1,E)=\{(e_1,\{b,d\})\}$ then $Cl_s^{\xi}((C,E)\tilde{\sqcap}(C_1,E))=\{(e_1,\{d\})\}$. But $Cl_s^{\xi}(C,E)=1_E$ and $Cl_s^{\xi}(C_1,E)=1_E$, then $Cl_s^{\xi}(C,E)\tilde{\sqcap}Cl_s^{\xi}(C_1,E)=1_E$. Implies that $Cl_s^{\xi}((C,E)\tilde{\sqcap}(C_1,E))\neq Cl_s^{\xi}(C,E)\tilde{\sqcap}Cl_s^{\xi}(C_1,E)$.

Corollary.4.12 For every soft subset (C,E) of $(U,\tilde{\tau},E)$, the following are true;

1. $1_E\backslash Cl_s^{\xi}(C,E)=Int_s^{\xi}(1_E\backslash(C,E))$;
2. $1_E\backslash Int_s^{\xi}(C,E)=Cl_s^{\xi}(1_E\backslash(C,E)$;
3. $Int_s^{\xi}(C,E)=1_E\backslash Cl_s^{\xi}(1_E\backslash(C,E))$.

Proof. Follows directly frome their definitions..

Theorem.4.13 If (C,E) is a soft subset of $(U,\tilde{\tau},E)$, then $Int_s^{\delta}(C,E)\tilde{\sqsubseteq}Int_s^{\xi}(C,E)\tilde{\sqsubseteq}Int_s(C,E)\tilde{\sqsubseteq}(C,E)\tilde{\sqsubseteq}$ $Cl_s(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)\tilde{\sqsubseteq}Cl_s^{\delta}(C,E)$.

Proof. Coming from their definitions.

Theorem.4.14 If (C,E) is a soft subset of $(U,\tilde{\tau},E)$, then we get the following,

1. $Int_s^{\xi}\left(Int_s(C,E)\right)=Int_s\left(Int_s^{\xi}(C,E)\right)=Int_s^{\xi}(C,E)$;
2. $Int_s^{\xi}\left(Int_s^{\delta}(C,E)\right)=Int_s^{\delta}\left(Int_s^{\xi}(C,E)\right)=Int_s^{\delta}(C,E)$;
3. $Cl_s^{\xi}\left(Cl_s(C,E)\right)=Cl_s\left(Cl_s^{\xi}(C,E)\right)=Cl_s^{\xi}(C,E)$;
4. $Cl_s^{\xi}\left(Cl_s^{\delta}(C,E)\right)=Cl_s^{\delta}\left(Cl_s^{\xi}(C,E)\right)Cl_s^{\delta}\ (C,E)$.

Proof. Followes directly from their definitions.

Theorem.4.15 If (A,E) is either soft open or a soft semi-closed subset of $(U,\tilde{\tau},E)$, and $(C,E)\tilde{\sqsubseteq}(A,E)$, then $\left(Cl_s^{\xi}\right)_A(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)$ where $\left(Cl_s^{\xi}\right)_A(C,E)$ denotes the soft ξ-closure of (C,E) in (A,E).

Proof. Suppose $e_x\tilde{\notin}Cl_s^{\xi}(C,E)$, then there exists a soft ξ-open set (G,E) in $(U,\tilde{\tau},E)$ containing e_x such that $(G,E)\tilde{\sqcap}(C,E)=0_E$. Since $(C,E)\tilde{\sqsubseteq}(A,E)$ (A,E), then $(G,E)\tilde{\sqcap}(A,E)\tilde{\sqcap}(C,E)=0_E$ .Put $(G_1,E)=(G,E)\tilde{\sqcap}(A,E)$. Since $(G,E)\tilde{\in}S\xi O(U,E)$ and (A,E) is either soft open or soft semi-closed subset of (U,E), then we have $(G_1,E)=(G,E)\tilde{\sqcap}(A,E)\tilde{\in}S\xi O(A,E)$ and $(G_1,E)\tilde{\sqcap}(C,E)=0_E$. Thus, $e_x\tilde{\notin}Cl_s^{\xi}(C,E)$. Therefore, we obtain $\left(Cl_s^{\xi}\right)_A(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)$..

Theorem.4.16 In $(U,\tilde{\tau},E)$ if $(A,E)\tilde{\in}SRO(U,E)$ and $(C,E)\tilde{\in}(A,E)\tilde{\sqsubseteq}(U,\tilde{\tau},E)$, then $Cl_s^{\xi}(C,E)\tilde{\sqcap}(A,E)$ $(A,E)=(Cl_s^{\xi})_A(C,E)$.

Proof. From (Theorem .4.15), $\left(Cl_s^{\xi}\right)_A(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)$ and $\left(\mathrm{Cl}_s^{\xi}\right)_A(\mathrm{C},\mathrm{E})\tilde{\sqsubseteq}(\mathrm{A},\mathrm{E})$ then

$$\left(Cl_s^{\xi}\right)_A(C,E)\tilde{\sqsubseteq}Cl_s^{\xi}(C,E)\tilde{\sqcap}(A,E) \qquad (3)$$

Now, let $e_x\tilde{\in}Cl_s^{\xi}(C,E)\tilde{\sqcap}(A,E)$, then $e_x\tilde{\in}Cl_s^{\xi}(C,E)$ and $e_x\tilde{\in}(A,E)$
$e_x\tilde{\in}Cl_s^{\xi}(C,E)$ this implies that $\forall\,\xi-$ $open\ set$ $(G,E)\tilde{\in}(U,\tilde{\tau},E)$ containing e_x such that $(G,E)\tilde{\sqcap}(C,E)\neq\phi$. Since $(A,E)\tilde{\in}SRO(U,E)$, then by (Theorem.3.12), $(G,E)\tilde{\sqcap}(A,E)$ is a soft ξ-open set in (A,E) and $(G,E)\tilde{\sqcap}(A,E)\tilde{\sqcap}(C,E)\neq\emptyset$
$e_x\tilde{\in}\left(Cl_s^{\xi}\right)_A(C,E)$
(4) (4)
fom (3) and (4), we obtain that
$\left(Cl_s^{\xi}\right)_A(C,E)=Cl_s^{\xi}(C,E)\tilde{\sqcap}(A,E)$.

Definition.4.17 Assume the soft topological space $(U,\tilde{\tau},E)$ and $e_x\tilde{\in}SP(U,E)$. If there is a soft ξ-open set (G,E) such that $e_x\tilde{\in}(G,E)$, then (G,E) is named soft ξ-open neighborhood (briefly, $nbds_s^{\xi}$) of e_x. The soft ξ-open neighborhood system of a soft element e_x, denoted by

$N_{\tilde{\tau}_\xi}(e_x)$ wich is $nbds_s^\xi(e_x) = \{(G,E):(G,E) \tilde{\in} S\xi O(U,E): e_x \tilde{\in} (G,E)\}$ and is named the family of any soft ξ-neighborhoods contained in it.

A soft subset (G,E) of $(U,\tilde{\tau},E)$ is named a soft ξ-open neighbourhood of a soft subset (C,E) of $(U,\tilde{\tau},E)$, denoted by $nbds_s^\xi(C,E)$ if there exists $(G_1,E) \tilde{\in} S\xi O(U,E)$ such that $(C,E) \tilde{\sqsubseteq} (G_1,E) \tilde{\sqsubseteq} (G, E)$.

Proposition.4.18 Assume the soft topological space $(U,\tilde{\tau},E)$. Then, the soft ξ-neighborhood system $N_{\tilde{\tau}_\xi}(e_x)$ at $e_x$in $(U,\tilde{\tau},E)$ has the following properties:

1. If $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$ then $e_x \tilde{\in} (C_1,E)$,
2. If $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$ and $(C_1,E) \tilde{\sqsubseteq} (C_2, E)$, then $(C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$,
3. If $(C_1,E),(C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$, then $(C_1,E) \tilde{\sqcap} (C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$
4. If $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$, then there exists a $(C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$ such that $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e'_y)$ for each $e'_y \tilde{\in} (C_2,E)$.

Proof. 1) If $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$, then there exists a soft ξ-open set (G,E) such that $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C_1,E)$. Therefore, we have $e_x \tilde{\in} (C_1,E)$.

2) Let $(C,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$ and $(C_1,E) \tilde{\sqsubseteq} (C_2,E)$. Since $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$, then there exists a soft ξ-open set (G,E) such that $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C_1,E)$. Therefore, we have $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C_1,E) \tilde{\sqsubseteq} (C_2,E)$

3) If $(C_1,E),(C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$ then there exist $(G_1,E),(G_2,E) \tilde{\in} S\xi O(U,E)$, such that $e_x \tilde{\in} (G_1,E) \tilde{\sqsubseteq} (C_1,E)$ and $e_x \tilde{\in} (G_2,E) \tilde{\sqsubseteq} (C_2,E)$. Hence, $e_x \tilde{\in} (G_1,E) \tilde{\sqcap} (G_2,E) \tilde{\sqsubseteq} (C_1,E) \tilde{\sqcap} (C_2,E)$. Since $(G_1,E) \tilde{\sqcap} (G_2,E)$ is a soft ξ-open set. We have $(C_1,E) \tilde{\sqcap} (C_2,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$.

4) If $(C_1,E) \tilde{\in} N_{\tilde{\tau}_\xi}(e_x)$, then there exists a soft ξ- open set (G,E) such that $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C_1,E)$. Put $(G_1,E) = (G,E)$. Then for every $e'_y \tilde{\in} (G_1,E)$, $e'_y \tilde{\in} (G_1,E) \tilde{\sqsubseteq} (G,E) \tilde{\sqsubseteq} (C_1,E)$. This implies that $e'_y \tilde{\in} (C_1,E)$.

Definition.4.19 Consider the soft topological space $(U,\tilde{\tau},E)$, and let $(C,E) \tilde{\sqsubseteq} (G,E)$, and $e_x \tilde{\in} (G,E)$. If all soft ξ-neighborhood of e_x soft intersects (C,E) in some soft elements except e_x itself, then e_x is named a soft ξ-limit point of (C,E). The set of all soft ξ-limit points of (C,E) is named soft ξ-drived set and is symbolized by $D_s^\xi(C,E)$.

Proposition.4.20 Assume the soft topological space $(U,\tilde{\tau},E)$ and (C,E) is a soft subset of $(U,\tilde{\tau},E)$, then $D_s(C,E) \tilde{\sqsubseteq} D_s^\xi(C,E)$.

Proof. Suppose $e_x \tilde{\in} D_s(C,E)$ and let (G,E) be any soft ξ-open set of $(U,\tilde{\tau},E)$ such that $e_x \tilde{\in} (G,E)$. Then, (G,E) is a soft open (by Definition 3.1). Hence, $(G,E) \tilde{\sqcap} (C,E)/e_x \neq 0_E$ This implies that, $e_x \tilde{\in} D_s^\xi(C,E)$. Therefore, $D_s(C,E) \tilde{\sqsubseteq} D_s^\xi(C,E)$.

The opposite of the (Proposition.4.21) generally may not be true as is displaied in (Example.4.22).

Example.4.21 Assume that $(U,\tilde{\tau},E)$ is a soft topological space as defined in the (Example.4.4). If we choose $(C,E) = \{(e_1,\{a,d\})\}$, so we get$D_s(C,E) = \{(e_1,\{a,d\})\}$ and $D_s^\xi(C,E) = 1_E$. Hence, $D_s^\xi(C,E\ ) \tilde{\not\sqsubseteq} D_s(C,E)$.

Some properties of soft ξ-drived set are mentioned in the following result:

Theorem.4.22 Assume that (C,E) and (C_1,E) be soft subsets of $(U,\tilde{\tau},E)$,. Then we have some properties as followe:

1. $D_s^\xi(0_E) = 0_E$,
2. $e_x \tilde{\in} D_s^\xi(C,E) \Rightarrow e_x \tilde{\in} D_s^\xi((C,E)\backslash e_x)$;
3. $D_s^\xi(C,E) \tilde{\sqcup} D_s^\xi(C_1,E) D_s^\xi((C,E) \tilde{\sqcup} (C_1,E))$,
4. $D_s^\xi((C,E) \tilde{\sqcap} (C_1,E)) \tilde{\sqsubseteq} D_s^\xi(C,E) \tilde{\sqcap} D_s^\xi(C_1,E)$,
5. $D_s^\xi\left(D_s^\xi(C,E)\right)\backslash(C,E) \tilde{\sqsubseteq} D_s^\xi(C,E)$,
6. $D_s^\xi((C,E) \tilde{\sqcup} D_s^\xi(C,E)) \tilde{\sqsubseteq} (C,E) \tilde{\sqcup} D_s^\xi(C,E$.

In general, $D_s^\xi((C,E) \tilde{\sqcap} (C_1,E)) \neq D_s^\xi(C,E) \tilde{\sqcap} D_s^\xi(C_1,E)$, as is shown in the (Example.4.23 **)** :

Example.4.23 If we choose the $\tilde{\tau}$ as defined in (Example.4.4). Let we take $(C,E) = \{(e_1,\{a\})\}$ and $(C_1,E) = \{(e_1,\{b\})\}$, then $D_s^\xi(C,E) = \{(e_1,\{b,c\})\}$ and $D_s^\xi(C_1,E) = \{(e_1,\{a,c\})\}$. $D_s^\xi(C,E) \tilde{\sqcap} D_s^\xi(C_1,E) = \{(e_1,\{c\})\}$, but $D_s^\xi((C,E) \tilde{\sqcap} (C_1,E)) = 0_E$. Therefore ,$D_s^\xi(\ (C,E) \tilde{\sqcap} (G,E)) = 0_E$.
Hence,$D_s^\xi\ ((C,E) \tilde{\sqcap} (C_1,E)) \neq D_s^\xi(C,E) \tilde{\sqcap} D_s^\xi(C_1,E)$.

Theorem.4.24 Assume the soft topological space $(U,\tilde{\tau},E)$ and (C,E) is a soft subset of $(U,\tilde{\tau},E)$. Then $(C,E) \tilde{\sqcup} D_s^\xi(C,E)$ is a soft ξ-closed.

Proof. Let $e_x \tilde{\notin} (C,E) \tilde{\sqcup} D_s^\xi(C,E)$. This implies that $e_x \tilde{\notin} (C,E)$or $e_x \tilde{\notin} D_s^\xi(C,E)$. Since $e_x \tilde{\notin} D_s^\xi(C,E)$ then there exists a soft ξ-open set (G,E) of $(U,\tilde{\tau},E)$, which contains no soft element of (C,E) other than e_x but $e_x \tilde{\notin} (C,E)$. So (G,E) does not contain any soft element of (C,E), $\Rightarrow (G,E) \tilde{\sqsubseteq} 1_E\backslash(C,E)$. Again, (G,E) is a soft ξ-open set for its soft elements. But as (G,E) does not contain any soft element of (C,E), no soft element of (G,E) can be soft ξ-limit point of (C,E). Therefore, no soft element of (G,E) can belong to $D_s^\xi(C,E) \Rightarrow (G,E) \tilde{\sqsubseteq} 1_E\backslash D_s^\xi(C,E)$. Hence, $e_x \tilde{\in} (1_E\backslash(C,E)) \tilde{\sqcap} (1_E\backslash D_s^\xi(C,E)) = 1_E((C,E) \tilde{\sqcup} D_s^\xi(C,E))$. Therefore $(C,E) \tilde{\sqcup} D_s^\xi(C,E)$ is a soft ξ-closed set . Hence, $Cl_s^\xi(C,E) \tilde{\sqsubseteq} (C,E) \tilde{\sqcup} D_s^\xi(C,E)$.

From the previous theorem we can conclud that $Cl_s^\xi(C,E) = (C,E) \tilde{\sqcup} D_s^\xi(C,E)$ for all soft subset (C,E) of $(U,\tilde{\tau},E)$,. Hence, we have $(C,E) \tilde{\sqcup} D_s^\xi(C,E)$ as soft ξ-closed if and only if $Cl_s^\xi(C,E) = (C,E) \tilde{\sqcup} D_s^\xi(C,E)$.

Definition.4.25 Assume the soft topological space $(U,\tilde{\tau},E)$, then the soft ξ-exterior of (C,E) of $(U,\tilde{\tau},E)$ is symbolized by $Ext_s^\xi(C,E)$ and defined by $Ext_s^\xi(C,E) = Int_s^\xi((C,E)^c)$. Thus e_x is said to be a soft ξ-exterior point of (C,E), if there exists a soft ξ-open set (G,E) such that $e_x \tilde{\in} (G,E) \tilde{\sqsubseteq} (C,E)^c$. It is obviouse that $Ext_s^\xi(C,E)$ is the biggest soft ξ-open set contained in $(C,E)^c$.

Theorem.4.26 Assume that (C,E) and (C_1,E) are soft subsets of $(U,\tilde{\tau},E)$, then the following holde

1. $Ext_s^{\xi}((C,E)\,\tilde{\sqcup}\,(C_1,E)) = Ext_s^{\xi}(C,E)\,\tilde{\sqcap}\,Ext_s^{\xi}(C_1,E)$.
2. $Ext_s^{\xi}(C,E)\,\tilde{\sqcup}\,Ext_s^{\xi}(C_1,E) \tilde{\sqsubseteq} Ext_s^{\xi}((C,E)\,\tilde{\sqcap}\,(C_1,E))$.

The next example showes that the equivalence does not holds in (2) in general

Example.4.27 Let $U=\{h_1,h_2\}$, and $E=\{e_1,e_2\}$ and consider the soft topology $\tilde{\tau}=\{0_E,1_E,(G_1,E),(G_2,E),(G_3,E)$ where $(G_1,E)=\{(e_1,U),(e_2,\{h_1\})\}$, $(G_2,E)=\{(e_1,\{h_2\}),(e_2,U)\}$, $(G_3,E)=\{(e_1,\{h_2\}),(e_2,\{h_1\})\}$ be defined on U. If we choose $(C,E)=\{(e_1,\{h_1\}),(e_2,\{h_2\})\}$, then

$$Ext_s^{\xi}(G_3,E)\,\tilde{\sqcup}\,Ext_s^{\xi}(C,E)$$
$$= Int_s^{\xi}(G_3^c,E)\,\tilde{\sqcup}\,Int_s^{\xi}(C^c,E)$$
$$= 0_E\,\tilde{\sqcup}\,(G_3,E) = (G_3,E)$$

but

$$Ext_s^{\xi}(G_3,E)\,\tilde{\sqcap}\,Ext_s^{\xi}(C,E) = Int_s^{\xi}((G_3,E)\,\tilde{\sqcap}\,(C,E))^c =$$
$$Int_s^{\xi}((G_3^c,E)\,\tilde{\sqcup}\,(C^c,E))$$
$$= Int_s^{\xi}((C,E)\,\tilde{\sqcup}\,(G_3,E))$$
$$= Int_s^{\xi(}\{(e_1,U),(e_2,U)\})$$
$$= \{(e_1,U),(e_2,U)\} \neq (G_3,E)$$

Definition.4.28

Assume the soft topological space $(U,\tilde{\tau},E)$, then soft ξ-boundary of (C,E) of $(U,\tilde{\tau},E)$ is symbolized as $Bd_s^{\xi}(C,E)$ and is defined as $Bd_s^{\xi}(C,E)=(Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Ext_s^{\xi}(C,E))^c$.

Remark.4.29 From the (Definition.4.28**)** it conclude that the soft boundary of both soft sets (C,E) and $(C,E)^c$ are the same.

Remark.4.30 For any (C,E) of $(U,\tilde{\tau},E)$, $Bd_s(C,E) \tilde{\sqsubseteq} Bd_s^{\xi}(C,E)$. In general, the converse may not be true as shown in the following example.

Example.4.31 Consider $(U,\tilde{\tau},E)$ as defined in (Example.4.4) If we take $(C,E)=\{(e_1,\{a,d\})\}$, then $Bd_s\{(e_1,\{a,d\})\}=\{(e_1,\{a\})\}$, but $Bd_s^{\xi}\{(e_1,\{a,d\})\}=\{(e_1,\{a,b,c\})\}$. Thus $Bd_s^{\xi}(C,E) \not\tilde{\sqsubseteq} Bd_s(C,E)$.

Remark.4.32 Assume the soft subsets (C,E) and (C_1,E) of $(U,\tilde{\tau},E)$. Then, $(C,E)\tilde{\sqsubseteq}(C_1,E)$ does not imply that either $Bd_s^{\xi}(C,E)\tilde{\sqsubseteq}Bd_s^{\xi}(C_1,E)$ or $Bd_s^{\xi}(C_1,E)\tilde{\sqsubseteq}Bd_s^{\xi}(C,E)$ as displaied in the following example.

Example.4.33 Consider $(U,\tilde{\tau},E)$ as defined in (Example 4.4). If we take $(C,E)=\{(e_1,\{b,c\})\}$ and $(C_1,E)=\{(e_1,\{a,b,c\})\}$, then $Bd_s^{\xi}(C,E)=\{(e_1,\{a, \ b,c\})\}$ but $Bd_s^{\xi}(C_1,E)=0_E$. This implies that $Bd_s^{\xi}(C,E) \not\tilde{\sqsubseteq} Bd_s^{\xi}(C_1,E)$.

Example.4.34 Assume $U=\{a,b,c,d\}$ and $E=\{e_1,e_2\}$ with the soft topology $\tilde{\tau}=\{0_E,1_E,\{(e_1,\{c\}),(e_2,\phi)\},\{(e_1,\{a,b\}),(e_2,\phi)\},\{(e_1,\{a,b,c\}),(e_2,\phi)\}\}$. Let $(C,E)=\{(e_1,\{c\}),(e_2,\phi)\}$ and $(C_1,E)=\{(e_1,\{a,c,d\}),(e_2,\emptyset)\}$, then $Bd_s^{\xi}(C,E)=\{(e_1,\{d\}),(e_2,\ \emptyset)\}$ but $Bd_s^{\xi}(C_1,E)=\{(e_1,\{a,b,d\}),\ (e_2,\emptyset)\}$ which shows that $Bd_s^{\xi}(C_1,E) \not\tilde{\sqsubseteq} Bd_s^{\xi}(C,E)$ with $(C,E)\tilde{\sqsubseteq}(C_1,E)$.

Theorem.4.35 Assume (C,E) is a soft subset of a soft space $(U,\tilde{\tau},E)$, then $Bd_s^{\xi}(C,E)=0_E$ if and only if (C,E) is both soft ξ-open and soft ξ-closed.

Proof. (Necessity) Assume that $Bd_s^{\xi}(C,E)=0_E$, then $Cl_s^{\xi}(C,E)\backslash Int_s^{\xi}(C,E);=0_E$, which implies that $Cl_s^{\xi}(C,E)=Int_s^{\xi}(C,E)=(C,E)$. Therefore, $(C,E)$is both a soft ξ-open and a soft ξ-closed set.

(Sufficiency) If $(C,E)\tilde{\in} S\xi O(U,E)$ and soft ξ-closed set, then $(C,E)=Cl_s^{\xi}(C,E)=Int_s^{\xi}(C,E)$ and hence $Bd_s^{\xi}(C,E)=Cl_s^{\xi}(C,E)\backslash Int_s^{\xi}(C,E)=0_E$.

Theorem.4.36 Assume the soft topological space $(U,\tilde{\tau},E)$, and (C,E) is a soft subset of $(U,\tilde{\tau},E)$. Then the statements in following hold

1. $Cl_s^{\xi}(C,E)=Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Bd_s^{\xi}(C,E)$,
2. $Bd_s^{\xi}(C,E)=Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,Cl_s^{\xi}(C,E)^c=Cl_s^{\xi}(C,E)\backslash Int_s^{\xi}(C,E)$,
3. $(Bd_s^{\xi}(C,E))^c=Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Int_s^{\xi}(C,E)^c=Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Ext_s^{\xi}(C,E)$,
4. $Int_s^{\xi}(C,E)=(C,E)\backslash Bd_s^{\xi}(C,E)$.

Proof. 1)

$$Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Bd_s^{\xi}(C,E)$$
$$= Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,(Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,Cl_s^{\xi}(C,E)^c)$$
$$= \left(Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Cl_s^{\xi}(C,E)\right)\tilde{\sqcap}\left(Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Cl_s^{\xi}(C,E)^c\right)$$
$$= Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,(Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,(Int_s^{\xi}(C,E))^c)$$
$$= Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,U$$
$$= Cl_s^{\xi}(C,E)$$

2)

$$Bd_s^{\xi}(C,E)=Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,Cl_s^{\xi}(C,E)^c$$
$$= Cl_s^{\xi}(C,E)\backslash Int_s^{\xi}(C,E)$$
$$= Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\left(Int_s^{\xi}(C,E)\right)^c$$
$$= Cl_s^{\xi}(C,E)\,\tilde{\sqcap}\,Cl_s^{\xi}(C,E)^c$$

3)

$$Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Cl_s^{\xi}(C,E)^c$$
$$= ((Int_s^{\xi}(C,E))^c)^c\,\tilde{\sqcup}\,(Int_s^{\xi}(C,E)^c)^c$$
$$= [\left(Int_s^{\xi}(C,E)\right)^c\,\tilde{\sqcap}\,Int_s^{\xi}(C,E)^c]^c$$
$$= [Cl_s^{\xi}(C,E)^c\,\tilde{\sqcap}\,Cl_s^{\xi}(C,E)]^c$$
$$= [Bd_s^{\xi}(C,E)]^c$$

4)

$$(C,E)\backslash Bd_s^{\xi}(C,E)=(C,E)\,\tilde{\sqcap}\,Bd_s^{\xi}(C,E)^c$$
$$= (C,E)\,\tilde{\sqcap}\,(Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,Int_s^{\xi}(C,E)^c) \quad \text{(by (1))}$$
$$= \left[(C,E)\,\tilde{\sqcap}\,Int_s^{\xi}(C,E)\right]\tilde{\sqcup}\left[(C,E)\,\tilde{\sqcap}\,Int_s^{\xi}(C,E)^c\right]$$
$$= Int_s^{\xi}(C,E)\,\tilde{\sqcup}\,0_E$$
$$= Int_s^{\xi}(C,E)$$

Theorem.4.37 Assume the soft subset (C,E) of $(U,\tilde{\tau},E)$, then

1. (C,E) in $(U,\tilde{\tau},E)$ is soft $\xi - open$ set if and only if $(C,E) \tilde{\sqcap} Bd_s^{\xi}(C,E) = 0_E$.
2. (C,E) is soft ξ-closed in $(U,\tilde{\tau},E)$ if and only if $Bd_s^{\xi}(C,E) \tilde{\sqsubseteq} (C,E)$.

Proof. 1) Suppose that (C,E) is a soft ξ-open set of $(U,\tilde{\tau},E)$, then $Int_s^{\xi}(C,E) = (C,E)$ implies $(C,E) \tilde{\sqcap} Bd_s^{\xi}(C,E) = Int_s^{\xi}(C,E) \tilde{\sqcap} Bd_s^{\xi}(C,E) = 0_E$.
Conversly, suppose that $(C,E) \tilde{\sqcap} Bd_s^{\xi}(C,E) = 0_E$. Then, $(C,E) \tilde{\sqcap} Cl_s^{\xi}(C,E) \tilde{\sqcap} Cl_s^{\xi}(C,E)^c = 0_E$. We have two cases:
First case,if

$$(C,E) \tilde{\sqcap} Cl_s^{\xi}(C,E)^c = 0_E$$
$$\Rightarrow (C,E) \tilde{\sqcap} \left(1_E \backslash Int_s^{\xi}(C,E)\right) = 0_E$$
$$\Rightarrow (C,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$$

$\Rightarrow (C,E)$ is soft ξ-open set
Second case, if

$$Cl_s^{\xi}(C,E) \tilde{\sqcap} Cl_s^{\xi}(C,E)^c = 0_E$$
$$\Rightarrow Cl_s^{\xi}(C,E) \tilde{\sqcap} (1_E \backslash Int_s^{\xi}(C,E) = 0_E$$
$$\Rightarrow Cl_s^{\xi}(C,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$$
$$\Rightarrow (C,E) \tilde{\sqsubseteq} Int_s^{\xi}(C,E)$$

$\Rightarrow (C,E)$ is soft ξ-open.

Therefore, (C,E) is soft ξ-open set.

2) Consider that (C,E) as a soft ξ-closed set in $(U,\tilde{\tau},E)$. Then $Cl_s^{\xi}(C,E) = (C,E)$. Now, $Bd_s^{\xi}(C,E) \tilde{\sqsubseteq} Cl_s^{\xi}(C,E) \tilde{\sqcap} Cl_s^{\xi}(C,E)^c \tilde{\sqsubseteq} Cl_s^{\xi}(C,E)$ that is, $Bd_s^{\xi}(C,E) \tilde{\sqsubseteq} (C,E)$.
Conversly, $Bd_s^{\xi}(C,E) \tilde{\sqsubseteq} (C,E)$. Then $Bd_s^{\xi}(C,E) \tilde{\sqcap} (C,E)^c = 0_E$. Since $Bd_s^{\xi}(C,E) = Bd_s^{\xi}(C,E)^c = 0_E$, we have $Bd_s^{\xi}(C,E)^c \tilde{\sqcap} (C,E)^c = 0_E$ By (1), $(C,E)^c$ is soft ξ-open and hence (C,E) is soft ξ-closed.

5 Soft ξ-continuous and soft almostξ-continuous

Definition.5.1 A soft mapping $f_{pu}: SS(U,E) \to SS(Y,A)$ is called a soft ξ-continuous(resp. soft almost ξ-continuous) at a soft element $e_x \tilde{\in} (U,E)$, if for each soft open nieghborhood set (G,A) in (Y,A) containing $f_{pu}(e_x)$, there exists a soft ξ-open neighberhood $(G_1,E) \tilde{\in} (U,E)$ containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,A)$ (resp, $f_{pu}(G_1,E) \tilde{\sqsubseteq} Int_s^{\xi}(Cl_s^{\xi}(G,A))$). If f_{pu} is soft ξ- continuous (resp., soft almost ξ-*continuous*) at every soft element in (U,E), then it is called "soft ξ-continuous"(resp., soft almost ξ-continuous).

Remark.5.2 Any soft ξ-continuous is soft continuous, but the opposing may not be true in general as is presented in the next example.

Example.5.3 Assume that $U = \{h_1,h_2\} = Y$, and $E = \{e_1,e_2\} = A$, Let the soft topology be defined as follows, where $\tilde{\tau} = \{0_E, 1_E(G,E)\}$ where $(G,E) = \{(e_1,\{h_1\}),(e_2,\{h_2\})\} = (G_1,A)$ and suppose that the soft map $f_{pu}: SS(U,E) \to SS(Y,A)$, where $u: U \to Y$ and $p: E \to A$ be identity functions, then f_{pu} is soft continuous function which is not soft ξ-continuous because $(G_1,A)$is a soft open set in (Y,A) containing the soft element $f_{pu}(e_x) = \{(e_1,\{h_1\}),(e_2,\phi),$, but there exists no soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,A)$.

Theorem.5.4 Any soft ξ-continuous function is soft θ-continuous
Proof. Suppose $f_{pu}: SS(U,E) \to SS(Y,A)$ is a soft ξ-continuous function and let $e_x \tilde{\in} (U,E)$and (G,A)be a soft open neighborhood set in (Y,A) containing $f_{pu}(e_x)$, then by definition of soft ξ-continuous there exists a soft open ξ-nieghborhood set (G_1,E) in (U,E) containing e_x and $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,E)$, then $Cl_s^{\xi}(f_{pu}(G_1,E)) \tilde{\sqsubseteq} Cl_s^{\xi}(G,A)$, since f_{pu} is soft ξ-continuous, then it is soft continuousm $f_{pu}(Cl_s^{\xi}(G_1,E)) \tilde{\sqsubseteq} Cl_s^{\xi}(f_{pu}(G_1,E)) \tilde{\sqsubseteq} Cl_s^{\xi}(G,A)$, this implies that $f_{pu}(Cl_s^{\xi}(G_1,E)) \tilde{\sqsubseteq} Cl_s^{\xi}(G,A)$. Therefore f_{pu} is a soft θ-continuous
or
Since any soft ξ-continuous is soft continuous, and every soft continuous is soft θ-continuous

Remark.5.5 Every soft ξ-continuous function is soft δ-continuous function, but the convers may not true in general as shown in the (Example.5.6).

Example.5.6 Consider $U = \{h_1,h_2\} = Y$ and $E = \{e_1,e_2\} = A$. Let $\tilde{\tau}_E$ be defined on U as $\tilde{\tau}_E = \{0_E, 1_E, \{(e_1,\phi),(e_2,\{h_1\})\}, \{(e_1,\phi),(e_2,\{h_2\})\},$ $\{(e_1,\emptyset),(e_2,U)\}$ and $\tilde{\tau}_A$ be defined on Y as $\tilde{\tau}_A = \{0_A, 1_A, \{(e_1,h_1),(e_2,h_1)\}$, then
$SC(U,E) = \{0_E, 1_E, \{(e_1,U),(e_2,\{h_2\})\},$ $\{(e_1,U),(e_2,\{h_1\})\}, \{(e_1,U),(e_2,\emptyset)\}$ *and* $SC(Y,A) = \{0_A, 1_A, \{(e_1,h_2),(e_2,\{h_2\})\}\}$. Let the soft function $f_{pu}: SS(U,E) \to SS(Y,A)$ be defined as follows, $u(h_1) = h_2$, $u(h_2) = h_1$, and $p(e_1) = e_1$, $p(e_2) = e_2$.
$SRO(U,E) = 0_E, 1_E, \{(e_1,\phi),(e_2,\{h_1\})\},$ $\{(e_1,\phi),(e_2,\{h_2\})\} = S\delta O(U,E)$
$SRO(Y,A) = 0_E, 1_E = SRO(Y,A)$.
The invers image of each soft regular open set in $SS(Y,A)$ is soft δ-open set in $SS(U,E)$, implies that f_{pu} is considered to be soft δ-continuous function.
$S\xi O(U,E) = \tilde{\tau}_E$.

$$f_{pu}^{-1}\{(e_1,\{h_1\}),(e_2,\{h_1\})\} = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$$

which is not soft ξ-open set in (U,E). Therefor, $f_{pu}: SS(U,E) \to SS(Y,A)$ is not soft ξ-continuous function.

Corollary.5.7 Any soft ξ-continuous function is a soft *semi*-continuous function.

Theorem.5.8 If $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft continuous and soft open function and (G,A) is a soft ξ-open set of (Y,A), then $f_{pu}^{-1}(G,A)$ is soft ξ-open set of (U,E).

Proof. Suppose (G,A) is a soft ξ-open set of (Y,A), then (G,A) is a soft open set of (Y,A) and $(G,A) = \sqcup_{\alpha\in\Delta} \{(H_\alpha,A): (H_\alpha,A)$ is a soft *smi* -closed set of (Y,A) for each $\alpha\}$. Then $f_{pu}^{-1}(G,A) = f_{pu}^{-1}(\sqcup_{\alpha\in\Delta} (H_\alpha,A)) = \sqcup_{\alpha\in\Delta} f_{pu}^{-1}(H_\alpha,A)$. Since f_{pu} is soft continuous, then $f_{pu}^{-1}(G,A)$ is a soft open set in (U,E). Also since $f_{pu}^{-1}(H_\alpha,A)$ is soft *semi* -closed in (U,E) and $f_{pu}^{-1}(G,A) = \sqcup_{\alpha\in\Delta} f_{pu}^{-1}(H_\alpha,A)$. Therefore, $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E).

Corollary.5.9 If $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft continuous and soft open function and (H,A) is a soft ξ-closed set of (Y,A), then $f_{pu}^{-1}(H,A)$ is a soft ξ-closed in (U,E).

Proof. Suppose that $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft continuous and soft open function and (H,A) is a soft ξ-closed set in (Y,A), then (H^c,A) is soft ξ-open in (Y,A). Since f_{pu} is soft open function, and each soft ξ-open set is soft open set, then $f_{pu}^{-1}(H^c,A)$ is soft ξ-open in (U,E). Hence, $f_{pu}^{-1}(H,A)$ is soft ξ-closed in (U,E).

Theorem.5.10 Consider the soft function $f_{pu}: SS(U,E) \to SS(Y,A)$. If f_{pu} is a soft ξ- continuous function if and only if the soft function f_{pu} is a soft continuous open function and for any soft element $e_x \tilde{\in} (U,E)$ and all soft open set (G,A) in (Y,A) containing $f_{pu}(e_x)$, there exists a soft semi-closed set $(H,E) \tilde{\in} (U,E)$ containing e_x such that $f_{pu}(H,E) \tilde{\sqsubseteq} (G,A)$.

Proof. (Necessity). Suppose $e_x \tilde{\in} (U,E)$ and (G,A) be any soft open set in (Y,A) containing $f_{pu}(e_x)$. Since f_{pu} is a soft ξ-continuous, there exists a soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,A)$. Since (G_1,E) is a soft ξ-open set, then for any $e_x \tilde{\in} (G_1,E)$, there exists a soft semi-closed set (H,E)in (U,E) such that $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} (G_1,E)$. Therefore, we have $f_{pu}(H,E) \tilde{\sqsubseteq} (G,A)$. Since f_{pu} is considered as a soft ξ-continuous, then f_{pu} is soft continuous.
(Sufficiency). Suppose (G,A) is any soft open set in (Y,A). Since f_{pu} is soft continuous and soft open, then $f_{pu}^{-1}(G,A)$ is a soft open set in (U,E). Suppose $e_x \tilde{\in} f_{pu}^{-1}(G,A)$. Then $f_{pu}(e_x) \tilde{\in} (G,A)$. By supposition, there eists a soft semi-closed set $(H,E) \tilde{\in} (U,E)$ soft containing e_x such that $f_{pu}(H,E) \tilde{\sqsubseteq} (G,A)$, which implies that $e_x \tilde{\in} (H,E) \tilde{\sqsubseteq} f_{pu}^{-1}(G,A)$. Therefore, $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E). Thus, $f_{pu}$is soft ξ-continuous

Theorem.5.11 For a soft function $f_{pu}: SS(U,E) \to SS(Y,A)$, the statements in what follow are equvalents,

1. f_{pu} is a soft ξ-continuous,
2. $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E) for any soft open set (G,A) in (Y,A),
3. $f_{pu}^{-1}(G,A)$ is a soft ξ-closed set in (U,E) for every soft closed set (G,A) in (Y,A),
4. $f_{pu}(Cl_s^{\xi}(C,E)) \tilde{\sqsubseteq} Cl_s(f_{pu}(C,E))$ for every soft subset (C,E) of (U,E),
5. $Cl_s^{\xi}(f_{pu}^{-1}(C_1,A)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_s(C_1,A))$ for every soft subset (C_1,A) of (Y,A),
6. $f_{pu}^{-1}(Int_s(C_1,A)) \tilde{\sqsubseteq} Int_s^{\xi}(f_{pu}^{-1}(C_1,A))$ for every soft subset (C_1,A) of (Y,A),
7. If f_{pu} is bijective soft function. Then, $Int_s(f_{pu}(C,E)) \tilde{\sqsubseteq} f_{pu}(Int_s^{\xi}(C,E))$for every soft subset (C,E) in (U,E).

Proof. (1)⟹ (2) Suppose (G,A) is any soft open set in (Y,A) and let $e_x \tilde{\in} f_{pu}^{-1}(G,A)$, then $f_{pu}(e_x) \tilde{\in} (G,A)$. By (1), there exists a soft ξ-open set (G_1,E) of (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,A)$. This implies that $e_x \tilde{\in} (G_1,E) \tilde{\sqsubseteq} f_{pu}^{-1}(G,A)$. Therefore, $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E).

(2)⟹(3) Consider (H,A) is any soft closed closed set of (Y,A). Then $(Y,A)\backslash(H,A)$ is a soft open set of (Y,A). By (2), $f_{pu}^{-1}\big((Y,A)\backslash(H,A)\big) = (U,E)\backslash f_{pu}^{-1}(H,A)$ is a soft ξ-open set in (U,E) and hence $f_{pu}^{-1}(H,A)$ is a soft ξ-closed set in (U,E).

(3)⟹(4) Suppse (C,E) is any soft subset of (U,E), then $f_{pu}(C,E) \tilde{\sqsubseteq} Cl_s(f_{pu}(C,E))$ and $Cl_s(f_{pu}(C,E))$ is a soft closed set in (Y,A). Hence, $(C,E) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_s\left(f_{pu}(C,E)\right))$. By (3), we have $f_{pu}^{-1}(Cl_s\left(f_{pu}(C,E)\right))$ is a soft ξ-closed set in (U,E). Therefore, $Cl_s^{\xi}(C,E) \tilde{\sqsubseteq} f_{pu}^{-1}\ (Cl_s\left(f_{pu}(C,E)\right))$. Hence, $f_{pu}(Cl_s^{\xi}(C,E)) \tilde{\sqsubseteq} Cl_s\ (f_{pu}(C,E))$.

(4)⟹(5) Suppose (C,A) is any soft subset of (Y,A), then $f_{pu}^{-1}(C,A)$ is a soft subset of (U,E). By (4), we have $f_{pu}\left(Cl_s^{\xi}\left(f_{pu}^{-1}(C,A)\right)\right) \tilde{\sqsubseteq} (f_{pu}((f_{pu}^{-1}(C,A)) = Cl_s(C,A)$. Hence, $Cl_s^{\xi}(f_{pu}^{-1}(C,A)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_s(C,A))$.

(5)⟹(6) Suppose (C_1,A) is any soft subset of (Y,A), then if we apply (5) to $(Y,A)\backslash(C_1,A)$, we obtain $Cl_s^{\xi}(f_{pu}^{-1}(Y,A)\backslash(C_1,A)) \tilde{\sqsubseteq} f_{pu}^{-1}(Cl_s((Y,A)\backslash(C_1,A))$ if and only if $Cl_s^{\xi}((U,E)\backslash f_{pu}^{-1}(C_1,A)) \tilde{\sqsubseteq} f_{pu}^{-1}((Y,A)\backslash Int_s^{\xi}(C_1,A))$ if and only if $(U,E)\backslash Int_s^{\xi}(f_{pu}^{-1}(C_1,A)) \tilde{\sqsubseteq} (U,E)\backslash f_{pu}^{-1}(Int_s(C_1,A))$, if and only if $f_{pu}^{-1}(Int_s(C_1,A)) \tilde{\sqsubseteq} Int_s^{\xi}(f_{pu}^{-1}(C_1,A))$. Therefore, $f_{pu}^{-1}(Int_s(C_1,A)) \tilde{\sqsubseteq} Int_s^{\xi}(f_{pu}^{-1}(C_1,A))$.

(6)⟹(7) Suppose (C,E) is any soft subset of (U,E), then $f_{pu}(C,E)$ is a soft subset of (Y,A). By (6), we have $f_{pu}^{-1}(Int_s\left(f_{pu}(C,E)\right)) \tilde{\sqsubseteq} Int_s^{\xi}(f_{pu}^{-1}\left(f_{pu}(C,E)\right)) = Int_s^{\xi}(C,E)$. Therefore, $Int_s(f_{pu}(C,E)) \tilde{\sqsubseteq} f_{pu}(Int_s^{\xi}(C,E)))$.

(7)⟹(1) Consider $e_x \tilde{\in} (U,E)$ and (G,A) is any soft open set of (Y,A) containing $f_{pu}(e_x)$, then $e_x \tilde{\in} f_{pu}^{-1}(G,A)$ and $f_{pu}^{-1}(G,A)$ is a soft subset of (U,E). By (7), we have $Int_s(f_{pu}\left(f_{pu}^{-1}(G,A)\right)) \tilde{\sqsubseteq} f_{pu}(Int_s^{\xi}\left(f_{pu}^{-1}(G,A)\right))$, then $Int_s(G,A) \tilde{\sqsubseteq} f_{pu}(Int_s^{\xi}\left(f_{pu}^{-1}(G,A)\right))$. Since (G,A) is a soft open set. Then, $(G,A) \tilde{\sqsubseteq} f_{pu}(Int_s^{\xi}\left(f_{pu}^{-1}(G,A)\right))$ which implies that $f_{pu}^{-1}(G,A) \tilde{\sqsubseteq} Int_s^{\xi}(f_{pu}^{-1}(G,A))$.

Theorem.5.12 For a soft function $f_{pu}: SS(U,E) \to SS(Y,A)$, the statement in the following are equvalent

1. f_{pu} is soft almost ξ-continuous,
2. For any $e_x \tilde{\in} (U,E)$ and any soft open set (G,A) of (Y,A) containing $f_{pu}(e_x)$, there exists a soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} Cl_s^{\xi}(G,A)$,
3. For any $e_x \tilde{\in} (U,E)$ and any soft regular open set (G,A) of (Y,A) containing $f_{pu}(e_x)$, there exists a soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\sqsubseteq} (G,A)$,
4. For any $e_x \tilde{\in} (U,E)$ and each soft δ-open set

(G,A) of (Y,A) containing $f_{pu}(e_x)$, there exists a soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\subseteq} (G,A)$.

Theorem.513 For f_{pu}: SS(U, E) → SS(Y, A), the statements in the following are equvalent:

1. f_{pu} is soft almost ξ-continuous,
2. $f_{pu}^{-1}(Int_s(Cl_s(G,A)))$ is soft ξ-open set in (U,E) for any soft open set (G,A) in (Y,A),
3. $f_{pu}^{-1}(Cl_s(Int_s(H,A)))$ is a soft ξ-closed set in (U,E) for any soft closed set (H,A) in (Y,A),
4. $f_{pu}^{-1}(H,A)$ is a soft ξ-closed set in (U,E) for any soft regular closed set (H,A) of (Y,A),
5. $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E) for any soft regular open set (G,A) of (Y,A).

Proof. (1) $\Longrightarrow$ (2): Suppose that (G,A) is any soft open set in (Y,A). Let $e_x \tilde{\in} f_{pu}^{-1}(Int_s(Cl_s(G,A)))$, then $f_{pu}(e_x) \tilde{\in} Int_s(Cl_s(G,A))$, and $Int_s(Cl_s(G,A))$ is a soft regular open set in (Y,A). Since f_{pu} is soft almost ξ-continuous then there exists a soft ξ-open set (G_1,E) in (U,E) containing e_x such that $f_{pu}(G_1,E) \tilde{\subseteq} Int_s^{\xi}(Cl_s^{\xi}(G,A))$. This implies that $e_x \tilde{\in} (G_1,E) \tilde{\subseteq} f_{pu}^{-1}(Int_s^{\xi}\left(Cl_s^{\xi}(G,A)\right))$. Therefore, $(Int_s^{\xi}\left(Cl_s^{\xi}(G,A)\right))$ is a soft ξ-open set in (U,E).

(2) $\Longrightarrow$(3): Suppose that (H,A) is a soft closed set in (Y,A). Then $(Y,A)\backslash(H,A)$ is a soft open set in (Y,A). By (2), $f_{pu}^{-1}(Int_s(Cl_s(Y,A)\backslash(H,A)))$. is a soft ξ-open set in (U,E) and $f_{pu}^{-1}(Int_s(Cl_s(Y,A)\backslash(H,A))) = f_{pu}^{-1}(Int_s(Y,A)\backslash Int_s(H,A) = f_{pu}^{-1}(Y,A)\backslash Cl_s(Int_s(H,A))) = (U,E)\backslash f_{pu}^{-1}(Cl_s(Int_s(H,A)))$ is a soft ξ-open set in (U,E) and hence $f_{pu}^{-1}(Cl_s(Int_s(H,A)))$ is a soft ξ-closed set in (U,E).

(3) $\Longrightarrow$(4): Suppose (H,A) is any soft regular closed set of (Y,A) then (H,A) is a soft closed set of (Y,A). By (3), $f_{pu}^{-1}(Cl_s(Int_s(H,A)))$is a soft ξ-closed set in (U,E). Since (H,A) is a soft regular close set, then $f_{pu}^{-1}\left(Cl_s(Int_s(H,A))\right) = f_{pu}^{-1}(H,A)$. Therefore, $f_{pu}^{-1}(H,A)$ is a soft ξ-closed set in (U,E).

(4) $\Longrightarrow$(5): Suppose (G,A) is a soft regular open set of (Y,A). Then $(Y,A)\backslash(G,A)$ is soft regular closed set in (Y,A) and by (4), we have $f_{pu}^{-1}((Y,A)\backslash(G,A)) = (U,E)\backslash f_{pu}^{-1}(G,A)$ as a soft ξ-closed in (U,E) and hence $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E).

(5) $\Longrightarrow$(1): Suppose $e_x \tilde{\in} (U,E)$ and let (G,A) be a soft regular open set in (Y,A) containing $f_{pu}(e_x)$. Then $e_x \tilde{\in} (G,A)$. By (5), we have $f_{pu}^{-1}(G,A)$ is a soft ξ-open set in (U,E). Therefore, we obtain $f_{pu}(f_{pu}^{-1}(G,E)) \tilde{\subseteq} (G,A)$. Hence, f_{pu} is soft almost ξ-continuous function.

Theorem.5.14 If $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft almost continuous, then the statements in the following are equvalent:

1. f_{pu} is soft almost ξ-continuous,
2. For any $e_x \tilde{\in} (U,E)$ and all soft open set (G,A) of (Y,A) containing $f_{pu}(e_x)$, there exists a soft *semi*-closed set (H,E) in (U,E) containing e_x such that $f_{pu}(H,E) \tilde{\subseteq} Int_s(Cl_s(G,A)) = Cl_s^S(G,A)$,
3. For any $e_x \tilde{\in} (U,E)$ and all soft open set (G,A) of (Y,A) containing $f_{pu}(e_x)$, there exists a soft *semi*-closed set (H,E) in (U,E) containing e_x such that $f_{pu}(H,E) \tilde{\subseteq} Cl_s^S(G,A)$,
4. For each $e_x \tilde{\in} (U,E)$ and every soft regular open set (G,A) in (Y,A) containing $f_{pu}(e_x)$, there exists a soft *semi*-closed (H,E) in (U,E) containing e_x such that $f_{pu}(H,E) \tilde{\subseteq} (G,A)$,
5. For any $e_x \tilde{\in} (U,E)$ and all soft δ-open set (G,A) in (Y,A)containing $f_{pu}(e_x)$, there exists a soft *semi*-closed (H,E) in (U,E) containing e_x such that $f_{pu}(H,E) \tilde{\subseteq} (G,A)$.

Theorem.5.15 Assume $f_{pu}: SS(U,E) \to SS(Y,A)$ is soft ξ-continuous(resp., soft almost ξ-continuous) function. If (G,E) is either a soft open(resp., soft regular open) subset of (U,E), then $f_{pu}/(G,E): SS(G,E) \to SS(Y,A)$ is soft ξ-continuous (resp., soft almost ξ-continuous) in the soft subspace (G,E).

Proof. Let (G_1,A) be every soft open (resp., soft regular open) set of (Y,A) containing $f_{pu}(e_x)$. Since f_{pu} is soft ξ-continuous (resp., soft almost ξ-continuous), then $f_{pu}^{-1}(G_1,A)$ is a soft ξ-open set or soft regular open subset of (U,E), $(f_{pu}\ /(G_1,A))^{-1}$
$(G,E) = f_{pu}^{-1}(G_1,A) \tilde{\cap} (G,E)$ is a soft ξ-open subspace of (G,E). Implies that $f_{pu}/\ (G,E): SS(G,E) \to SS(Y,A)$ is considered as soft ξ-continuous (resp., soft almost ξ-continuous) in soft subspace (U,E).

Theorem.5.16 If for any $e_x \tilde{\in} (U,E)$, there exists a soft regular open set (G,E) of (U,E) containing e_x such that $f_{pu}/\ (G,E): SS(G,E) \to SS(Y,A)$ is soft ξ-continuous (resp., soft almost ξ-continuous). Then the soft function $f_{pu}: SS(U,E) \to SS(Y,A)$ is a soft ξ-continuous (resp., soft almost ξ-continuous) function.

Proof. Suppose that $e_x \tilde{\in} (U,E)$, then by supposition, there exists a soft regular open set (G,E) containing e_x such that $f_{pu}/\ (G,E): SS(G,E) \to SS(Y,A)$ is soft ξ-continuous (resp., soft almost ξ-continuous). Let (G_1,A) be any soft open set of (Y,A) containing $f_{pu}(e_x)$, there exists a soft ξ-open set (G_2,E) in (G_1,E) containing e_x such that $(f_{pu}/\ (G_1,E))(G_2,E) \tilde{\subseteq} (G,A)$ (resp., $(f_{pu}/\ (G_1,E))\ (G_2,E) \tilde{\subseteq} Int_s(Cl_s(G,A)))$. Since (G_1,E) is a soft set, then (G_1,E) is a soft ξ-open subspace in (U,E) and hence $f_{pu}(G_2,E) \tilde{\subseteq} (G,A)$ (resp., $f_{pu}(G_2,E) \tilde{\subseteq} Int_s(Cl_s(G,A))$. This shows that f_{pu} is soft ξ-continuous (resp., soft almost ξ-continuous).

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