RESIDUAL POWER SERIES METHOD FOR SOLVING KLEIN-GORDON SCHRÖDINGER EQUATION

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ABSTRACT:
In this work, the Residual Power Series Method(RPSM) is used to find the approximate solutions of Klein Gordon Schrödinger (KGS) Equation. Furthermore, to show the accuracy and the efficiency of the presented method, we compare the obtained approximate solution of Klein Gordon Schrödinger equation by Residual Power Series Method(RPSM) numerically and graphically with the exact solution.

KEYWORDS: Residual Power Series Method, Klein Gordon Schrödinger.

1. INTRODUCTION

The Residual Power Series Method (RPSM) was first presented in 2013 by Abu Arqub [4]. The RPSM is a very significant method for finding the numerical solution of linear and nonlinear differential equations. This method has been applied successfully by many authors to find the numerical solutions for numerous problems, Such as, Alquran, M. (2014) in [2], El-Ajou, A., Arqub, O. A., & Momani, S. (2015) in [5], Alquran, M. (2015) in [3], and Kumar, S., Kumar, A., & Baleanu, D. (2016) in [7].

In addition to that, the RPSM has been used by Abu Arqub, O., El-Ajou, A., Bataineh, A. S., & Hashim, I.(2013) to find the solution of generalized Lane-Emden equations in [1] and it has been applied in 2016 by İnç, M., Korpinar, Z. S., Al Qurashi, M. M., & Baleanu, D. to find the approximate solutions of some nonlinear equations[6]. It was also applied by Manaa, S. A.,& Mosa, N. M. in 2019 to solve the Kaup-Boussinesq System[8] and by Modanli, M., Abdulazeez, S. T., & Husien, A. M. in 2020 to solve "pseudo hyperbolic partial differential equations with nonlocal conditions"[9].

In this paper, the Residual Power Series Method (RPSM) [1],[6],[8] and [9] is applied to solve Klein-Gordon Schrödinger (KGS) Equation. Moreover, we got all results and figures by using Mathematica program.

2. MATHEMATICAL MODEL

Consider the Klein-Gordon Schrödinger (KGS) Equation[10]

$$\begin{cases} iq_t = -aq_{xx} - b\varphi q, \\ \varphi_{tt} = c^2\varphi_{xx} - \beta^2\varphi + \lambda|q|^2 \end{cases} \quad (1)$$

Where $x \in \mathbb{R}, t \geq 0$, $i = \sqrt{-1}$ and a, b, c, β and λ are considered to be the arbitrary constants. While, $\boldsymbol{q}$ is the complex nucleon-fields and $\boldsymbol{\varphi}$ is the neutral real meson-fields. Moreover, the system (1) has wide-range applications in many fields such as quantum physics and modern physics [10]-[12].

Equation (1) can be separate into real part and imaginary part. Then, we will get a tripled system (a system of three real equations) as follow:

$$\begin{cases} u_t = -av_{xx} - bv\varphi, \\ v_t = au_{xx} + bu\varphi, \\ \varphi_{tt} = c^2\varphi_{xx} - \beta^2\varphi + \lambda u^2 + \lambda v^2. \end{cases} \quad (2)$$

For $q = u + iv$. (3)

Where u and v and φ are real functions of x and t.

3. BASIC IDEA OF RESIDUAL POWER SERIES METHOD

Considering the general form of nonlinear partial differential equation:

$$LU(x,t) = R[U(x,t)] + N[U(x,t)]. \quad (4)$$

Subject to the initial conditions:

$$U(x,0) = f(x). \quad (5)$$

Where $L = \frac{\partial^m}{\partial t^m}$, $m \in \mathbb{N}$, is the highest order partial derivative with respect to time t. While, the reminder linear term is R, the nonlinear operator is N.

Now, the solution of Equations (4) by RPSM around the initial point t=0 is written as a power series [1],[6],[8] and [9] as follow:

$$U(x,t) = \sum_{i=0}^{\infty} f_i(x)t^i. \quad (6)$$

Where $i = 0,1,2,...$, then, we can definite $U_n(x,t)$ to donate the n-th truncated series of $U(x,t)$, i.e.

$$U_n(x,t) = \sum_{i=0}^{n} f_i(x)t^i. \quad (7)$$

Where $U_0 = U(x,0)$ and $f(x) = f_0(x)$. (8)

Now, substituting equation (8) into equation (7) we get:

$$U_n(x,t) = U_0 + \sum_{i=1}^{n} f_i(x)t^i. \quad (9)$$

For $t \geq 0, x \in I, n = 1,2,....$

In order to evaluate the coefficients $f_i(x)$, for $i = 1,2,3,\ldots,n$ of equation (9), we first defined the residual function for (4), as:

$$ResU(x,t) = LU(x,t) - R[U(x,t)] - N[U(x,t)] \quad (10)$$

Then, the n-th residual function $ResU_n(x,t)$ is defined as follow:

$$ResU_n(x,t) = LU_n(x,t) - R[U_n(x,t)] - N[U_n(x,t)],$$
$$n = 1,2,\ldots \quad (11)$$

As Arqub, O. A. and his colleagues stated in [1], [4] and [5] that:

- $ResU(x,t) = 0$.
- $\lim\limits_{n\to\infty} ResU_n(x,t) = ResU(x,t)$, $\forall x \in I$, $t \geq 0$
- $\frac{\partial^m ResU_n(x,t)}{\partial t^m}\Big|_{t=0} = 0$, $m = 0,1,2,\ldots,n$. (12)

Therefore, we can obtain all required coefficients $f_i(x)$ (for all i) of the power series of Equation (4).

4. DERIVATION OF RPSM FOR SOLVING KGS-SYSTEM

Assume that equation(2) has the following Initial conditions:
$$u(x,0) = u_0 = f_0(x),$$
$$v(x,0) = v_0 = g_0(x), \quad (13)$$
$$\varphi(x,0) = \varphi_0 = h_0(x),$$
$$\varphi_t(x,0) = T_1(x).$$

Now, applying RPSM on equation (2) with equation (13). Then, the solution of equation (2) by RPSM around the initial point t=0 is written as:

$$u(x,t) = \sum_{i=0}^{\infty} f_i(x)t^i . \quad (14)$$

$$v(x,t) = \sum_{i=0}^{\infty} g_i(x)t^i . \quad (15)$$

$$\varphi(x,t) = \sum_{i=0}^{\infty} h_i(x)t^i . \quad (16)$$

Where $i = 0,1,2,\ldots$, then, we can definite $u_n(x,t)$, $v_n(x,t)$ and $\varphi_n(x,t)$ to give the n-th truncated series of $u(x,t), v(x,t)$ and $\varphi(x,t)$, i.e.

$$u_n(x,t) = \sum_{i=0}^{n} f_i(x)t^i . \quad (17)$$

$$v_n(x,t) = \sum_{i=0}^{n} g_i(x)t^i . \quad (18)$$

$$\varphi_n(x,t) = \sum_{i=0}^{n} h_i(x)t^i . \quad (19)$$

Now, substituting equation (13) into equations (17), (18) and (19) we get:

$$u_n(x,t) = f_0(x) + \sum_{i=1}^{n} f_i(x)t^i . \quad (20)$$

$$v_n(x,t) = g_0(x) + \sum_{i=1}^{n} g_i(x)t^i . \quad (21)$$

$$\varphi_n(x,t) = h_0(x) + T_1(x)t + \sum_{i=2}^{n} h_i(x)t^i . \quad (22)$$

In order to calculate the value of coefficients $f_i(x), g_i(x)$ and $h_i(x)$, of equations (20), (21) and (22), for all $i = 1,2,3,\ldots,n$. We defined the residual function for equation(2), as:

$$Resu(x,t) = u_t + av_{xx} + bv\varphi . \quad (23)$$
$$Resv(x,t) = v_t - au_{xx} - bu\varphi. \quad (24)$$
$$Res\varphi(x,t) = \varphi_{tt} - c^2\varphi_{xx} + \beta^2\varphi - \lambda u^2 - \lambda v^2. \quad (25)$$

Then, the n-th residual function $Resu_n(x,t), Resv_n(x,t)$ and $Res\varphi_n(x,t)$ is defined as follow:

$$Resu_n(x,t) = \frac{\partial u_n}{\partial t} + a\frac{\partial^2 v_n}{\partial x^2} + bv_n \varphi_n . \quad (26)$$

$$Resv_n(x,t) = \frac{\partial v_n}{\partial t} - a\frac{\partial^2 u_n}{\partial x^2} - bu_n \varphi_n. \quad (27)$$

$$Res\varphi_n(x,t) = \frac{\partial^2 \varphi_n}{\partial t^2} - c^2\frac{\partial^2 \varphi_n}{\partial x^2} + \beta^2\varphi_n - \lambda(u_n)^2 - \lambda(v_n)^2. \quad (28)$$

For equations (26) and (27) $n = 1,2,3,\ldots$ and for equations (28) $n = 2,3,\ldots$[9]. Then, we will have :

- $Resu(x,t) = 0$, $Resv(x,t) = 0$ and $Res\varphi(x,t) = 0$.
- $\lim\limits_{n\to\infty} Resu_n(x,t) = Resu(x,t)$, $\lim\limits_{n\to\infty} Resv_n(x,t) = Resv(x,t)$ and

$\lim\limits_{n\to\infty} Res\varphi_n(x,t) = Res\varphi(x,t)$, $\forall x \in I$, $t \geq 0$

- $\frac{\partial^m Resu_n(x,t)}{\partial t^m}\Big|_{t=0} = 0$, $\frac{\partial^m Resv_n(x,t)}{\partial t^m}\Big|_{t=0} = 0$ and

$\frac{\partial^m Res\varphi_n(x,t)}{\partial t^m}\Big|_{t=0} = 0$, $m = 0,1,2,\ldots,n$. (29)

Therefore, to find the solution of equation (2), we need evaluate all required coefficients $f_i(x)$, $g_i(x)$ and $h_i(x)$ (for all i).

For $n = 1$ substitute into equations (20), (21) and (22), then the first RPS approximate solution of equation (2) is :

$$u_1(x,t) = f_0(x) + f_1(x) t . \quad (30)$$
$$v_1(x,t) = g_0(x) + g_1(x) t . \quad (31)$$

Because, in (25) we have φ_{tt}, then[9],

$$\varphi_1 = h_0(x) + T_1(x) t . \quad (32)$$

From equations (26), (27) we get:

$$Resu_1(x,t) = \frac{\partial u_1}{\partial t} + a\frac{\partial^2 v_1}{\partial x^2} + bv_1 \varphi_1. \quad (33)$$

$$Resv_1(x,t) = \frac{\partial v_1}{\partial t} - a\frac{\partial^2 u_1}{\partial x^2} - bu_1 \varphi_1. \quad (34)$$

By substituting equations (30) and (31) into equations (33) and (34), we get the 1st residual functions as:

$$Resu_1(x,t) = f_1(x) + a(g''_0(x) + g''_1(x)t) + b(g_0(x) + g_1(x)t)(h_0(x) + h_1(x)t). \quad (35)$$
$$Resv_1(x,t) = g_1(x) - a(f''_0(x) + f''_1(x)t) - b(f_0(x) + f_1(x)t)(h_0(x) + h_1(x)t). \quad (36)$$

Applying equation (29) on (30) and (31) we have

$$Resu_1(x,t)|_{t=0} = f_1(x) + a(g''_0(x)) + b(g_0(x))(h_0(x))$$
$$= 0$$

$$Resv_1(x,t)|_{t=0} = g_1(x) - a(f''_0(x)) - b(f_0(x))(h_0(x))$$
$$= 0$$

Which leads to:

$$f_1(x) = -a(g''_0(x)) - b(g_0(x))(h_0(x)). \quad (37)$$

$$g_1(x) = a(f''_0(x)) + b(f_0(x))(h_0(x)). \quad (38)$$

We can rewrite equations (37) and (38) as:

$$f_1(x) = R_1(x). \quad (39)$$
$$g_1(x) = S_1(x). \quad (40)$$

For $R_1(x) = -a\left(g''_0(x)\right) - b(g_0(x))(h_0(x))$

and $S_1(x) = a\left(f''_0(x)\right) + b(f_0(x))(h_0(x))$

Again, substituting equations (39) and (40) into equations (30) and (31) respectively, with equation (32), we get the first approximate solutions of residual power series method(RPSM) of equation (2):

$$u_1(x,t) = f_0(x) + R_1(x)t,$$
$$v_1(x,t) = g_0(x) + S_1(x)t, \qquad (41)$$
$$\varphi_1(x,t) = h_0(x) + T_1(x)t.$$

Setting $n = 2$ in equations (20), (21) and (22), then the 2nd RPS approximate solution of equation (2) is:

$$u_2(x,t) = f_0(x) + R_1(x)t + f_2(x)t^2,$$
$$v_2(x,t) = g_0(x) + S_1(x)t + g_2(x)t^2, \qquad (42)$$
$$\varphi_2(x,t) = h_0(x) + T_1(x)t + h_2(x)t^2.$$

Then, the 2nd residual functions $Resu_2(x,t)$, $Resv_2(x,t)$ and $Res\varphi_2(x,t)$ is:

$$Resu_2(x,t) = \frac{\partial u_2}{\partial t} + a\frac{\partial^2 v_2}{\partial x^2} + bv_2\,\varphi_2,$$

$$Resv_2(x,t) = \frac{\partial v_2}{\partial t} - a\frac{\partial^2 u_2}{\partial x^2} - bu_2\,\varphi_2, \qquad (43)$$

$$Res\varphi_2(x,t) = \frac{\partial^2 \varphi_2}{\partial t^2} - c^2\frac{\partial^2 \varphi_2}{\partial x^2} + \beta^2\varphi_2 - \lambda(u_2)^2 - \lambda(v_2)^2.$$

By substituting equation (42) into equation (43), we get the 2st residual functions as:

$Resu_2(x,t) = R_1(x) + 2f_2(x)\,t + a(g''_0(x) + S''_1(x)t + g''_2(x)t^2) + b(g_0(x) + S_1(x)t + g_2(x)t^2)(h_0(x) + T_1(x)t + h_2(x)t^2),$

$Resv_2(x,t) = S_1(x) + 2g_2(x)\,t - a(f''_0(x) + R''_1(x)t + f''_2(x)t^2) - b(f_0(x) + R_1(x)t + f_2(x)t^2)(h_0(x) + T_1(x)t + h_2(x)t^2), \qquad (44)$

$Res\varphi_2(x,t) = 2h_2(x) - c^2(h''_0(x) + T''_1(x)t + h''_2(x)t^2) + \beta^2(h_0(x) + T_1(x)t + h_2(x)t^2) - \lambda(f_0(x) + R_1(x)t + f_2(x)t^2)^2 - \lambda(g_0(x) + S_1(x)t + g_2(x)t^2)^2.$

Applying equation (29) on (44) we get

$\frac{\partial Resu_2(x,t)}{\partial t}|_{t=0} = 2f_2(x) + aS''_1(x) + b(g_0(x)T_1(x) + S_1(x)h_0(x)) = 0$

$\frac{\partial Resv_2(x,t)}{\partial t}|_{t=0} = 2g_2(x) - aR''_1(x) - b(f_0(x)T_1(x) + R_1(x)h_0(x)) = 0$

$Res\varphi_2|_{t=0} = 2h_2(x) - c^2h''_0(x) + \beta^2h_0(x) - \lambda(f_0(x))^2 - \lambda(g_0(x))^2 = 0$

Which leads to:

$$f_2(x) = -\frac{1}{2}\left(aS''_1(x) + b(g_0(x)T_1(x) + S_1(x)h_0(x))\right),$$

$$g_2(x) = \frac{1}{2}\left(aR''_1(x) + b\binom{f_0(x)T_1(x) +}{R_1(x)h_0(x)}\right), \qquad (45)$$

$$h_2(x) = \frac{1}{2}\left(c^2h''_0(x) - \beta^2h_0(x) + \lambda(f_0(x))^2 + \lambda(g_0(x))^2\right).$$

We can rewrite equations (45) as:

$$f_2(x) = R_2(x),$$
$$g_2(x) = S_2(x), \qquad (46)$$
$$h_2(x) = T_2(x).$$

For

$$R_2(x) = -\frac{1}{2}\left(aS''_1(x) + b(g_0(x)T_1(x) + S_1(x)h_0(x))\right)$$

$$S_2(x) = \frac{1}{2}\left(aR''_1(x) + b(f_0(x)T_1(x) + R_1(x)h_0(x))\right)$$

And

$$T_2(x) = \frac{1}{2}\left(c^2h''_0(x) - \beta^2h_0(x) + \lambda(f_0(x))^2 + \lambda(g_0(x))^2\right)$$

Again, substituting (46) into (42), we get the 2nd approximate solutions of RPSM of equation (2):

$$u_2(x,t) = f_0(x) + R_1(x)t + R_2(x)\,t^2,$$
$$v_2(x,t) = g_0(x) + S_1(x)t + S_2(x)t^2, \qquad (47)$$
$$\varphi_2(x,t) = h_0(x) + T_1(x)t + T_2(x)t^2,$$

And so we can follow the same way for $n = 3,4,\cdots$, to find $f_3, f_4, f_5, \ldots, g_3, g_4, g_5, \ldots$ and $h_3, h_4, h_5, \ldots$. Then, the approximate solutions of RPSM can take the following forms:

$$u(x,t) = \sum_{i=0}^{\infty} f_i(x)t^i$$

$$v(x,t) = \sum_{i=0}^{\infty} g_i(x)t^i, \qquad (48)$$

$$\varphi(x,t) = \sum_{i=0}^{\infty} h_i(x)t^i.$$

5. APPLICATION WITH NUMERICAL RESULTS (TABLES, FIGURES)

This section will be devoted to find the numerical results (Tables, Figures) of Klein-Gordon Schrödinger (KGS) Equation by using RPSM.

Example: If we take the arbitrary constants of equations (1) and (2) to be: $\{b = c = \lambda = \beta = 1 \text{ and } a = \frac{1}{2}\}$ [11]. Then we get:

$$\begin{cases} iq_t = -\frac{1}{2}q_{xx} - \varphi q, \\ \varphi_{tt} = \varphi_{xx} - \varphi + |q|^2. \end{cases} \quad x \in \mathbb{R}, t \geq 0, \ i = \sqrt{-1} \quad (49)$$

and

$$\begin{cases} u_t = -\frac{1}{2}v_{xx} - v\varphi, \\ v_t = \frac{1}{2}u_{xx} + u\varphi, \\ \varphi_{tt} = \varphi_{xx} - \varphi + u^2 + v^2. \end{cases} \qquad (50)$$

The exact solitary wave solution of system (49) as in [11],[12] are:

$$\begin{cases} q(x,t) = \dfrac{3\text{Sech}^2[\frac{x - x_0 - t\alpha}{2\sqrt{1 - \alpha^2}}]\ e^{i(x\alpha + t(-\frac{\alpha^2}{2} + \frac{1}{2 - 2\alpha^2}))}}{2\sqrt{2 - 2\alpha^2}}, \\ \varphi(x,t) = -\dfrac{3\text{Sech}^2[\frac{x - x_0 - t\alpha}{2\sqrt{1 - \alpha^2}}]}{4(\alpha^2 - 1)}. \end{cases} \qquad (51)$$

With the initial conditions:

$$\begin{cases} q(x,0) = \dfrac{3\text{Sech}^2[\frac{x - x_0}{2\sqrt{1 - \alpha^2}}]\ e^{ix\alpha}}{2\sqrt{2 - 2\alpha^2}}, \\ \varphi(x,0) = -\dfrac{3\text{Sech}^2[\frac{x - x_0}{2\sqrt{1 - \alpha^2}}]}{4(\alpha^2 - 1)}. \end{cases} \qquad (52)$$

Therefore, the exact solitary wave solutions the system (50) are:

$$\begin{cases} u(x,t) = \dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0-t\alpha}{2\sqrt{1-\alpha^2}}\right]\mathrm{Cos}\left[x\alpha+t\left(-\frac{\alpha^2}{2}+\frac{1}{2-2\alpha^2}\right)\right]}{2\sqrt{2-2\alpha^2}}, \\[3mm] v(x,t) = \dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0-t\alpha}{2\sqrt{1-\alpha^2}}\right]\mathrm{Sin}\left[x\alpha+t\left(-\frac{\alpha^2}{2}+\frac{1}{2-2\alpha^2}\right)\right]}{2\sqrt{2-2\alpha^2}}, \quad (53) \\[3mm] \varphi(x,t) = -\dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0-t\alpha}{2\sqrt{1-\alpha^2}}\right]}{4(\alpha^2-1)}. \end{cases}$$
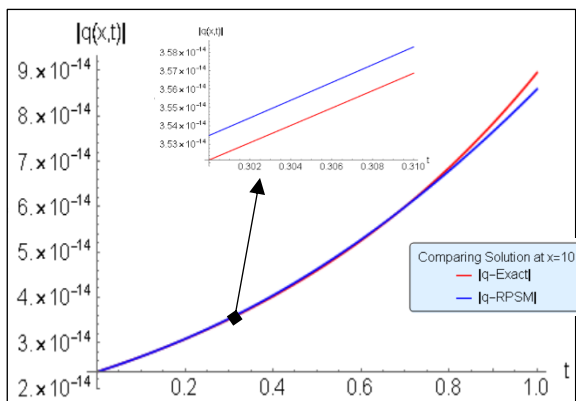
With the initial conditions

$$\begin{cases} u(x,0) = \dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0}{2\sqrt{1-\alpha^2}}\right]\mathrm{Cos}[x\alpha]}{2\sqrt{2-2\alpha^2}}, \\[3mm] v(x,0) = \dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0}{2\sqrt{1-\alpha^2}}\right]\mathrm{Sin}[x\alpha]}{2\sqrt{2-2\alpha^2}}, \quad (54) \\[3mm] \varphi(x,0) = -\dfrac{3\,\mathrm{Sech}^2\left[\frac{x-x_0}{2\sqrt{1-\alpha^2}}\right]}{4(\alpha^2-1)}. \end{cases}$$

Where x_0 is the initial phase and $|\alpha| > 0$ is the propagating velocity of the wave [11].
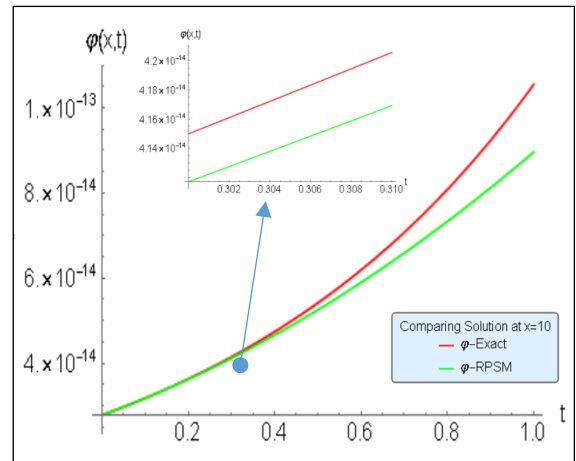
In the following table and figures we considered initial-values ($\alpha = 0.8, x_0 = -10$) [11].

Table 1. The absolute error between the exact solution and approximate solutions by RPSM of equation (49), $x = 10$ and $t \in [0,1]$.

| Time	$|qRPSM - qExact|$	$|\varphi RPSM - \varphi Exact|$
0	0	0
0.1	$2.03011067 * 10^{-17}$	$1.136640964 * 10^{-17}$
0.2	$1.67971031 * 10^{-16}$	$9.410909187 * 10^{-17}$
0.3	$5.864410163 * 10^{-16}$	$3.289471049 * 10^{-16}$
0.4	$1.4382761 * 10^{-15}$	$8.081125316 * 10^{-16}$
0.5	$2.907058086 * 10^{-15}$	$1.636992592 * 10^{-15}$
0.6	$5.199395215 * 10^{-15}$	$2.93600597 \times 10^{-15}$
0.7	$8.547065525 * 10^{-15}$	$4.842746766 * 10^{-15}$
0.8	$1.320930239 * 10^{-14}$	$7.514434237 * 10^{-15}$
0.9	$1.947523274 * 10^{-14}$	$1.113071195 * 10^{-14}$
1	$2.766648102 * 10^{-14}$	$1.589684618 * 10^{-14}$

Figure 2: The comparison between RPSM and the exact solution for $\varphi(x,t)$. when $x = 10$ and $t \in [0,1]$.
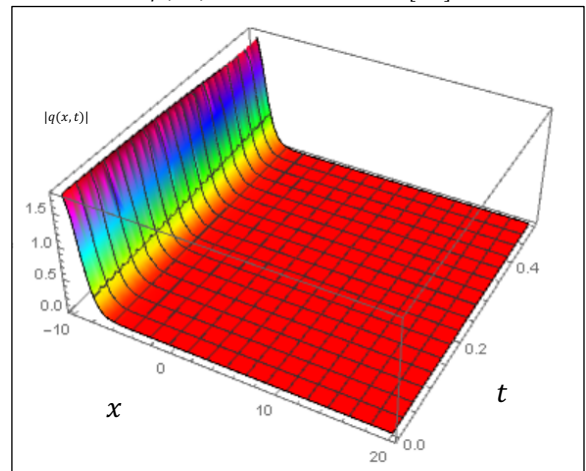
Figure 3: The surfaces of exact solutions $|q(x,t)|$, when $x \in [-10,20]$ and $t \in [0,0.5]$.

Figure 1: The comparison between RPSM and the exact solution for $|q(x,t)|$, when $x = 10$ and $t \in [0,1]$.
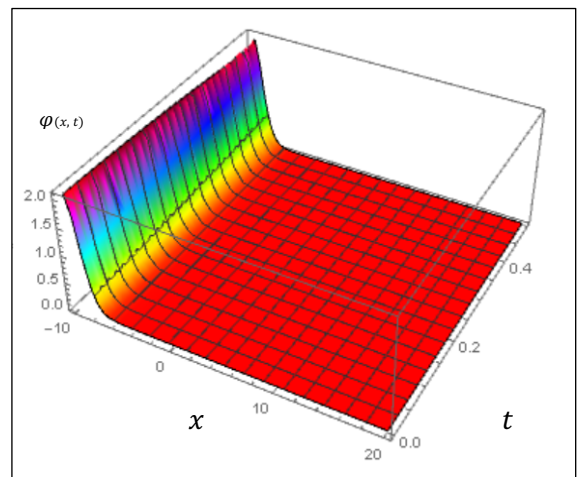
Figure 4: The surfaces of exact solutions $\varphi(x,t)$, when $x \in [-10,20]$ and $t \in [0,0.5]$.
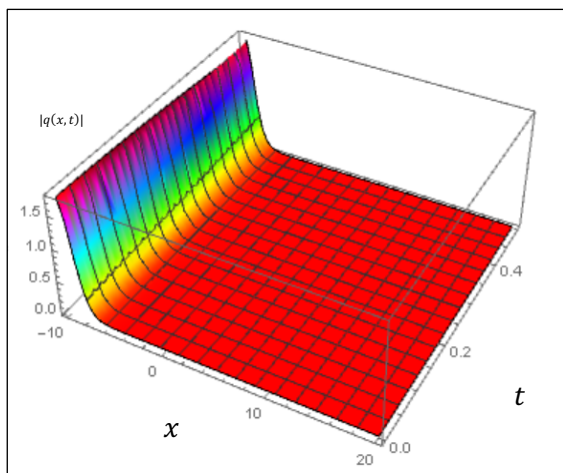
Figure 5: The surfaces of approximate solutions $|q(x,t)|$ by RPSM when $x \in [-10,20]$ and $t \in [0,0.5]$.
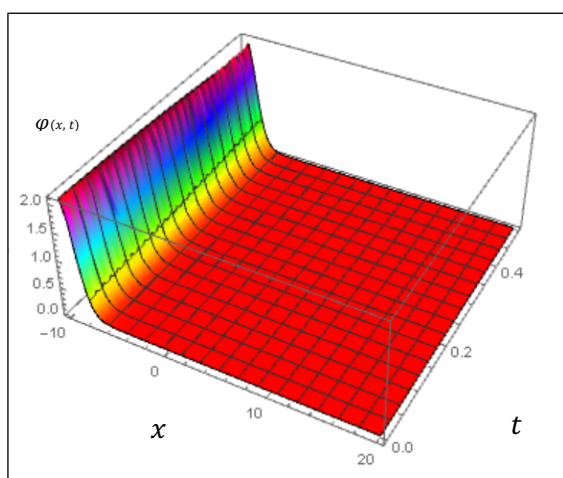

Figure 6: The surfaces of approximate solutions $\varphi(x,t)$ by RPSM when $x \in [-10,20]$ and $t \in [0,0.5]$.

6.   CONCLUSION

In this paper the Klein-Gordon Schrödinger (KGS) equation was solved numerically by using Residual Power Series Method. We took an example of (KGS) equation to find the comparison between our solution and the exact solution. Moreover, we concluded from Tabl.1 and Figures (1-6) that the RPSM is very accurate and effective in solving (KGS) equation.

7.   REFERENCES

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