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New Successive Approximation Methods for Solving Strongly Nonlinear Jaulent-Miodek Equations

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ABSTRACT:
In this paper, we propose two new techniques for solving system of nonlinear partial differential equations numerically, which we first combine Laplace transformation method into a successive approximation method. Second, we combine Padé [2,2] technique into the first proposed technique. To test the efficiency of our techniques, Jaulent-Miodek system was used, which contains partial differential equations and has strongly nonlinear terms. Experimental results revealed that the first proposed technique gives better results when the interval of t is small in terms of error approximation in tabular and graphical manners. Moreover, the results also demonstrated that the second proposed technique gives better results regardless of the given interval of t in terms of the least square errors.

KEYWORDS: Jaulent-Miodek, Successive Approximation Method, Laplace Transformation, Padé Technique

1. INTRODUCTION

The majority of scientific problems and occurrences are nonlinear in nature. Except for a few situations, mathematicians, physicists, and engineers are fascinated by nonlinear partial differential equations (NLPDEs). In most cases, accurate solutions to NLPDEs are unavailable. Nonlinear optics, hydrodynamics, plasma physics, radar and rheology, quantum physics, and optical fiber communication, among other domains, use differential equations. There has been a significant amount of research into linear and nonlinear PDE systems. For example, these models appear in the propagation of shallow water waves, ice streaming, the Brusselator model of the chemical reaction-diffusion model, granular matter theory, and several scientific domains such as mathematical biology, solid-state physics, and the fields described above. Because the majority of them lack accurate analytical answers, they must be solved using other approaches (Tracinà & Khalique, n.d.).
In this paper, the successive approximation method (SAM), and we propose two new techniques for solving strongly nonlinear Jaulent-Miodek **(JM)** system (Fan, 2003; Veeresha et al., 2019).

The Mathematical model of (JM) equation are:

$$u_t + u_{xxx} + \frac{3}{2}vv_{xxx} + \frac{9}{2}v_x v_{xx} - 6uu_x - 6uvv_x - \frac{3}{2}u_x v^2 = 0 \qquad (1)$$
$$v_t + v_{xxx} - 6u_x v - 6uv_x - \frac{15}{2}v_x v^2 = 0,$$

The exact solitary solution of Eq. (1) as in (Jalili et al., 2008) are:

$$u(x,t) = s - \frac{b*k*\text{sech}\big(k(Rt+x)\big)}{2} - \frac{3*c*\big(\text{sech}\big(k(Rt+x)\big)\big)^2}{4} \qquad (2)$$
$$v(x,t) = b + k*\text{sech}\big(k(Rt+x)\big)$$

Where s, b, k, R, and c are arbitrary constants, while $s = \frac{1}{4}(c - b^2)$, $k = \sqrt{c}$, and $R = \frac{1}{2}(b^2 + c)$
And initial conditions are:

$$u(x,0) = s - \frac{b*k*\text{sech}(kx)}{2} - \frac{3*c*(\text{sech}(kx))^2}{4} \qquad (3)$$
$$v(x,0) = b + k*\text{sech}(kx)$$

2. BASIC IDEA OF SUCCESSIVE APPROXIMATION METHOD

SAM is a method used to solve any initial value problem such as $u' = f(t,u)$, $u(x,t_0) = u_0$. This method can be expressed mathematically as shown in Eq. (7). For more details about the derivation of this method, the reader is referred to (Manaa et al., 2013). Recently, this method has attracted the attention of many authors such as (Hashem, 2015; Jafari, 2014) due to its simplicity and its outstanding results. Inspired by the aforementioned merits of SAM, we use it in our proposed techniques to be applied to the system of equations (1).
Suppose we have initial value problem:

$$u' = f(t,u),\ u(x,t_0) = u_0 \qquad (4)$$

SAM is used for solving Eq. (4) as follows:
Start with taking the integration for both sides of Eq. (4), to obtain a first approximation $u_1(x,t)$

$$u_1(x,t) = u_0(x,0) + \int_0^t u\big(\eta, u_0(x,\tau)\big)d\eta \qquad (5)$$

Then this $u_1(x,t)$ is substituted again in the integral of Eq. (5) to obtain a second approximation $u_2(x,t)$

$$u_2(x,t) = u_0(x,0) + \int_0^t u\big(\eta, u_1(x,\tau)\big)d\eta \qquad (6)$$

This process can be continued to obtain the n^{th} approximation or (the general form) of SAM can be written as:

$$u_{n+1}(x,t) = u_0(x,0) + \int_0^t u\big(\eta, u_n(x,\tau)\big)d\eta \qquad (7)$$

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3. LAPLACE TRANSFORMATION

There are many techniques when we apply Laplace Transformation with any other methods such as, Laplace Substitution Method (Mahavidyalaya, 2012), Laplace Decomposition Method (Mohamed & Torky, 2013), and the VIM-Laplace-Padé (Abassy et al., 2007), He-Laplace Method (Adam, 2015).

4. PADE TECHNIQUE

The main advantage of the Padé approximation over the Taylor series approximation is that the Taylor series approximation can oscillate, resulting in an approximation error bound. Furthermore, in a finite region, Taylor series approximations can never blow up. The Padé approximation is used to address these flaws. A ratio of two polynomials gives the Padé approximation of a function. Using the coefficients in the Tylor series expression of the function, the coefficients of the polynomial in both the numerator and denominator. The Padé approximation of a function, symbolized by $\left[\frac{P_M(x)}{Q_N(x)}\right]$ or for simplicity $\left[\frac{M}{N}\right]$, is a rational function defined by

$$\left[\frac{M}{N}\right] = \frac{p_0 + p_1 x + p_2 x^2 + \cdots + p_M x^M}{1 + q_1 x + q_2 x^2 + \cdots + q_N x^N}, \quad (8)$$

For further understanding this technique step by step, the reader is referred to (Sabali et al., 2018).

5. SAM WITH LAPLACE THEN PADE TECHNIQUE

Here, we use a different technique from that used by (Abassy et al., 2007), which includes the following steps:
Taking Laplace Transformation on both sides of equation (7)

$$L\big(u_{n+1}(x,t)\big) = L\left(u_0(x,0) + \int_0^t u\big(\eta, u_n(\eta,\tau)\big)d\eta\right) \quad (9)$$

$$L\big(u_{n+1}(x,t)\big) = u_0(x,0) + \frac{1}{s}\left\{L\left(\int_0^t u\big(\eta, u_n(\eta,\tau)\big)d\eta\right)\right\} \quad (10)$$

Taking Laplace inverse on both sides of equation (10), we get:

$$u_{n+1}(x,t) = u_0(x,0) + L^{-1}\left[\frac{1}{s}\left\{L\left(\int_0^t u\big(\eta, u_n(\eta,\tau)\big)d\eta\right)\right\}\right] \quad (11)$$

Now, solving equation (11) by usual SAM (such as Taking $n = 0, 1$)

$$u_2(x,t) = u_0(x,0) + L^{-1}\left[\frac{1}{s}\left\{L\left(\int_0^t u\big(\eta, u_1(\eta,\tau)\big)d\eta\right)\right\}\right] \quad (12)$$

Finally, Apply Padé [2,2] Technique of equation (12).

6. APPLICATION

In this section, we demonstrate the analysis of numerical methods such as SAM, SAM-Laplace Transformation, and SAM-Laplace-Padé Technique by applying them to the system of nonlinear partial differential Equations (1)

Note that we used *MATHEMATICA* software to obtain all numerical results using all of the above methods. This is due to its simplicity and powerful manipulation.

The numerical results of System (1) obtained by SAM using three approximate terms can be seen below:

$$u(x,t) = u_2(x,t)$$

$$u(x,t) = \frac{1}{16384}t^4(-35640b^7c^{7/2} - 17820b^5c^{9/2} - 2970b^3c^{11/2} - 165bc^{13/2} - 225504b^6c^4\text{Cosh}[\sqrt{c}x] \ldots$$

And

$$v(x,t) = v_2(x,t)$$

$$v(x,t) = \frac{1}{1024}t^4(19440b^6c^{7/2}\text{Cosh}[\sqrt{c}x] + 9720b^4c^{9/2}\text{Cosh}[\sqrt{c}x] + 1620b^2c^{11/2}\text{Cosh}[\sqrt{c}x] \ldots$$

Due to the very long resulting series obtained by SAM, this paper only refers to a few parts of the resulting series.
The numerical results of System (1) obtained by SAM-Laplace transformation using three approximate terms can be seen below:

$$u(x,t) = u_2(x,t)$$

$$u(x,t) = \frac{1}{4}(-b^2 + c - 2b\sqrt{c}\text{Sech}[\sqrt{c}x] - 3c\text{Sech}[\sqrt{c}x]^2) + \frac{1}{4}t^2(5bc^2\text{Sech}[\sqrt{c}x]^3\text{Tanh}[\sqrt{c}x] + 24c^{5/2}\text{Sech}[\sqrt{c}x]^4\text{Tanh}[\sqrt{c}x] - \ldots$$

And

$$v(x,t) = v_2(x,t)$$

$$v(x,t) = b + \sqrt{c}\text{Sech}[\sqrt{c}x] + \frac{1}{2}t^2\left(-5c^2\text{Sech}[\sqrt{c}x]^3\text{Tanh}[\sqrt{c}x] + c^2\text{Sech}[\sqrt{c}x]\text{Tanh}[\sqrt{c}x]^3\right) + \ldots$$

Again, due to the very long resulting series obtained by SAM-Laplace transformation, this paper only refers to a few parts of the resulting series.

The numerical results of System (1) obtained by SAM-Laplace-Padé $[2,2]$ technique using three approximate terms can be seen below:

$$u(x,t) = u_2(x,t)$$

$$u(x,t) = \frac{\frac{1}{4096}(-504b\sqrt{c} - 462b^2\text{Cosh}[\sqrt{c}x] - 1050c\text{Cosh}[\sqrt{c}x] - \ldots}{(1+(t\,(180\,b\,c\,\text{Sinh}[8\ x] + 30b^2\sqrt{c}\text{Sinh}[9\sqrt{c}x] - \ldots}$$

And

$$v(x,t) = v_2(x,t)$$

$$v(x,t) = \frac{(\frac{1}{128}(35b + 70\sqrt{c}\text{Cosh}[\sqrt{c}x] + 56b\text{Cosh}[2\sqrt{c}x] + \ldots}{1 + \frac{t(-2322b^2c^{3/2}\text{Cosh}[\sqrt{c}x] - 387c^{5/2}\text{Cosh}[\sqrt{c}x] -}{6c(43\text{Sinh}[\sqrt{c}x] + 63\text{Sinh}[3\sqrt{c}x] +} \ldots}$$

Again, due to the very long resulting series obtained by SAM-Laplace-Padé technique, this paper only refers to a few parts of the resulting series.

Tables 1 and 2 show the differences between exact and approximate solutions using **SAM, SAM-Laplace, and SAM-Laplace-Padé $[2,2]$ technique** for $u(x,t)$ and $v(x,t)$, respectively, when $x = 0.1$ [arbitrary chosen] and $t \in [\,0,1\,]$.

Table 1.

$u(x,t)$	Exact	SAM	SAM-Laplace	SAM-Laplace-Padé
$(0.1, 0.0)$	−0.00552 4225051	−0.00552 4225051	−0.00552 4225051	−0.00552 4225051
$(0.1, 0.2)$	−0.00552 4209319	−0.00552 4101465	−0.00552 4201029	−0.00552 420103
$(0.1, 0.4)$	−0.00552 4193429	−0.00552 3943504	−0.00552 4130614	−0.00552 4130652
$(0.1, 0.6)$	−0.00552 4177381	−0.00552 3751167	−0.00552 4015738	−0.00552 4016017
$(0.1, 0.8)$	−0.00552 4161175	−0.00552 3524454	−0.00552 3857575	−0.00552 3858721
$(0.1, 1.0)$	−0.00552 4144811	−0.00552 3263364	−0.00552 365651	−0.00552 3659914

Table 2.

$v(x,t)$	Exact	SAM	SAM-Laplace	SAM-Laplace-Padé
$(0.1, 0.0)$	0.10999 50002	0.10999 50002	0.10999 50002	0.10999 50002
$(0.1, 0.2)$	0.10999 48987	0.10999 4953	0.10999 49176	0.10999 49172
$(0.1, 0.4)$	0.10999 47962	0.10999 50233	0.10999 47361	0.10999 47273
$(0.1, 0.6)$	0.10999 46926	0.10999 52112	0.10999 4549	0.10999 4495
$(0.1, 0.8)$	0.10999 45881	0.10999 55166	0.10999 44403	0.10999 4257
$(0.1, 1.0)$	0.10999 44825	0.10999 59395	0.10999 44851	0.10999 40312

Again, Tables 3 and 4 show the absolute error between the exact solution and approximate solutions by **SAM, SAM-Laplace, and SAM-Laplace-Padé technique** for $u(x,t)$ and $v(x,t)$ respectively, when $x = 0.1$ and $t \in [0,1]$.

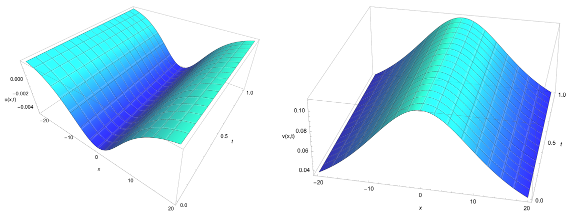
Table 3.

$u(x,t)$	| Exact – SAM |	| Exact – SAM, Laplace |	| Exact – SAM,Laplace,Padé |
$(0.1, 0.0)$	0	2.60209×10^{-18}	6.61797×10^{-16}
$(0.1, 0.2)$	1.07854×10^{-7}	8.28985×10^{-9}	8.28863×10^{-9}
$(0.1, 0.4)$	2.49925×10^{-7}	6.28147×10^{-8}	6.27768×10^{-8}
$(0.1, 0.6)$	4.26214×10^{-7}	1.61643×10^{-7}	1.61364×10^{-7}
$(0.1, 0.8)$	6.36721×10^{-7}	3.036×10^{-7}	3.02454×10^{-7}
$(0.1, 1.0)$	8.81447×10^{-7}	4.88301×10^{-7}	4.84897×10^{-7}
Least square error	$\mathbf{2.43015 \times 10^{-13}}$	$\mathbf{5.77274 \times 10^{-14}}$	$\mathbf{5.7129 \times 10^{-14}}$

Table 4.

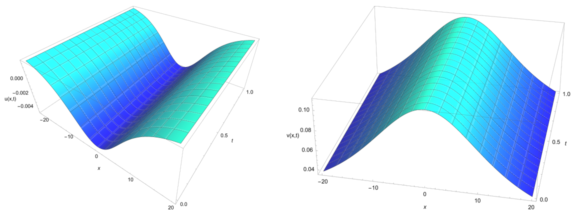
$v(x,t)$	| Exact – SAM |	| Exact – SAM, Laplace |	| Exact – SAM,Laplace,Padé |
$(0.1, 0.0)$	0	1.38778×10^{-17}	7.34135×10^{-15}
$(0.1, 0.2)$	1.07854×10^{-7}	1.88647×10^{-8}	1.85192×10^{-8}
$(0.1, 0.4)$	2.49925×10^{-7}	6.00576×10^{-8}	6.8925×10^{-8}
$(0.1, 0.6)$	4.26214×10^{-7}	1.43673×10^{-7}	1.97651×10^{-7}
$(0.1, 0.8)$	6.36721×10^{-7}	1.47818×10^{-7}	3.31063×10^{-7}
$(0.1, 1.0)$	8.81447×10^{-7}	2.58779×10^{-9}	4.51328×10^{-7}
Least square error	$\mathbf{5.3423 \times 10^{-13}}$	$\mathbf{9.38935 \times 10^{-15}}$	$\mathbf{6.00819 \times 10^{-14}}$

Also, the Fig. 2, Fig. 3, Fig. 4, and Fig. 5 below are the surfaces for the exact solution of **J.M** system, **SAM, SAM-Laplace, and SAM-Laplace-Padé**$[2,2]$ **technique**, respectively, when $x \in [-20, 20]$ and $t \in [0,1]$.



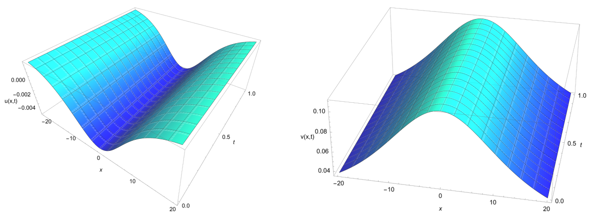
(a) $u(x,t)$ **(b)** $v(x,t)$

Fig. 2 The surfaces of **exact** solutions.



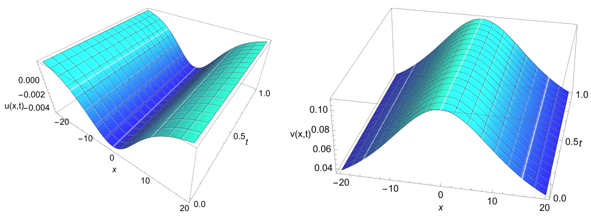
(a) $u(x,t)$ **(b)** $v(x,t)$

Fig. 3 The surfaces of **SAM** solutions.



(a) $u(x,t)$ **(b)** $v(x,t)$

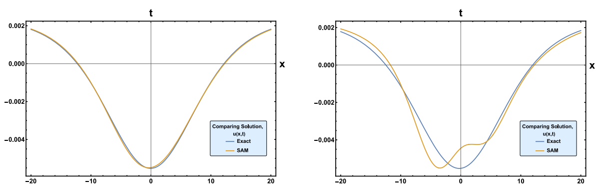
Fig. 4 The surfaces of **SAM-Laplace** solutions.



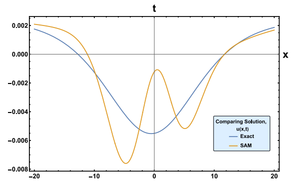
(a) $u(x,t)$ **(b)** $v(x,t)$

Fig. 5 The surfaces of **SAM-Laplace-Padé** solutions.

The Curves in Fig. 6, Fig. 7, Fig. 8, Fig. 9, Fig. 10, and Fig. 11 show us how the **SAM, SAM-Laplace, and SAM-Laplace-Padé** $[2,2]$ curves are close to the exact solution curve, when $x = \mathbf{0.1}$ and t has different values $t = \{10, 30, 50, 100\}$. The reason for that is to show how the method are stable when we increase the value of t, but as t increases, we see the SAM-Laplace curve blows up and diverges from the exact solution curve while SAM and SAM-Laplace-Padé curve preserves its path with the exact solution curve. Although the curves cannot give us the precise results as can be seen from Tables 3 and 4, the reason for this will be explained later.

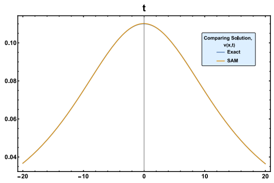


(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$ **(b)** $x \in [-20, 20]$ **and** $t \in [0, 50]$

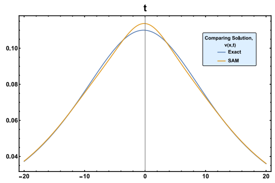


(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

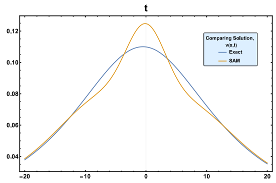
Fig. 6 The Curves of **SAM** for different values of t for $u(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$

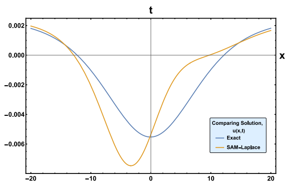


(b) $x \in [-20, 20]$ **and** $t \in [0, 50]$

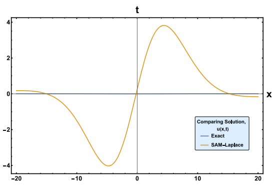


(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

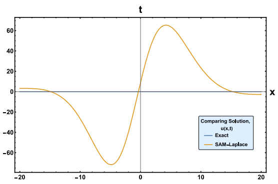
Fig. 7 The Curves of **SAM** for different values of t for $v(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$

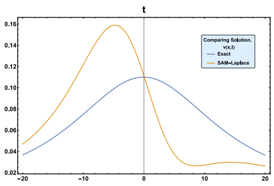


(b) $x \in [-20, 20]$ **and** $t \in [0, 50]$

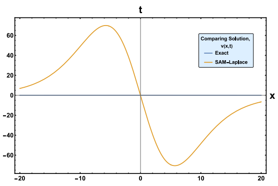


(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

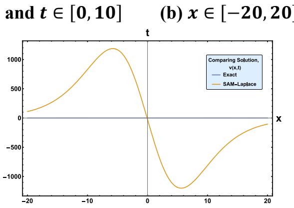
Fig. 8 The Curves of **SAM-Laplace** for different values of t for $u(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$

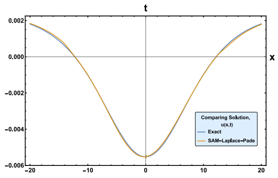


(b) $x \in [-20, 20]$ **and** $t \in [0, 50]$

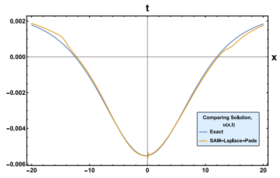


(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

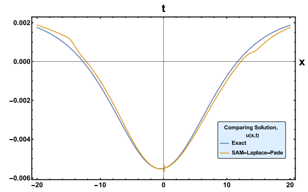
Fig. 9 The Curves of **SAM-Laplace** for different values of t for $v(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$

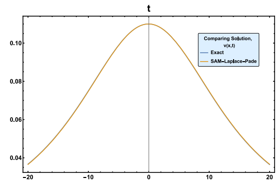


(b) $x \in [-20, 20]$ **and** $t \in [0, 50]$

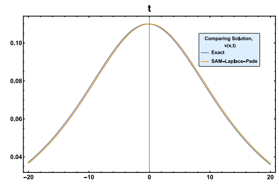


(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

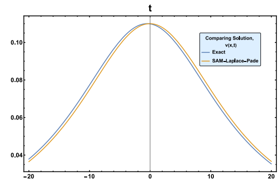
Fig. 10 The Curves of **SAM-Laplace-Padé** for different values of t for $u(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$



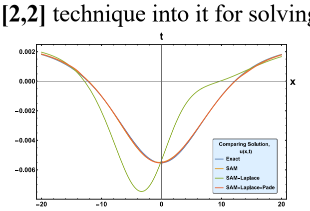
(b) $x \in [-20, 20]$ **and** $t \in [0, 50]$



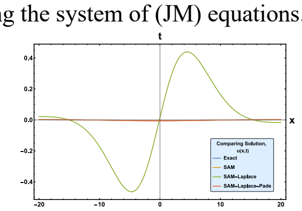
(c) $x \in [-20, 20]$ **and** $t \in [0, 100]$

Fig. 11 The Curves of **SAM-Laplace-Padé** for different values of t for $v(x,t)$

Again, the family curves in Fig. 12 and Fig. 13 show how the **SAM** and the **SAM-Laplace-Padé** $[2, 2]$ curves are close to the exact solution curve when $x = 0.1$ and t has different values $t = \{10, 30\}$, which indicate that the curves preserve their path with the exact solution curve. As its evident from the results, the **SAM-Laplace** method does not preserve its path with the exact solution curve. Thus, to overcome this flaw, we combine **Padé [2,2]** technique into it for solving the system of (JM) equations.

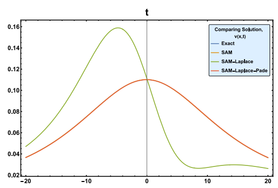


(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$

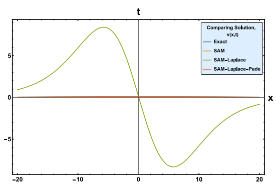


(b) $x \in [-20, 20]$ **and** $t \in [0, 30]$

Fig. 12 The Family Curves of **SAM, SAM-Laplace, SAM-Laplace-Padé** for different values of t for $u(x,t)$



(a) $x \in [-20, 20]$ **and** $t \in [0, 10]$



(b) $x \in [-20, 20]$ **and** $t \in [0, 30]$

Fig. 13 The Family Curves of **SAM, SAM-Laplace, SAM-Laplace-Padé** for different values of t for $v(x,t)$

Finally, to see the precise difference exactly when t increases, Tables 5 and 6 show the least square errors between **SAM, SAM-Laplace**, and **SAM-Laplace-Padé** $[2, 2]$ when $x = 0.1$ and $t \in [0, 100]$.

Table 5.

$u(x,t)$	| Exact – SAM |	| Exact – SAM, Laplace |	| Exact – SAM,Laplace,Padé |
Least square error	3.7729×10^{-4}	1.21518×10^{3}	2.91505×10^{-5}

Table 6.

$v(x,t)$	| Exact – SAM |	| Exact – SAM, Laplace |	| Exact – SAM,Laplace,Padé |
Least square error	4.39697×10^{-3}	3.52589×10^{4}	6.85448×10^{-7}

7. CONCLUSION

In this paper, a strongly nonlinear system of partial differential equations, which is well known as (JM) equation, was solved numerically by using Successive approximation method, Successive approximation with Laplace, and Successive approximation with **Laplace** then **Padé [2, 2]** technique. From the results obtained by Tables 1-4 and Figures 6(a), 7(a), 10(a), and 11(a), we can conclude that **SAM-Laplace** and **SAM-Laplace-Padé [2,2]** methods perform better than SAM when t is small in terms of least square error. Moreover, the results obtained from Tables 5-6 and Figures 6-13 indicate that with increasing t, **SAM-Laplace** deviates from the exact solution, while **SAM** and **SAM-Laplace-Padé [2,2]** keep their trajectory with the exact solution. However, **SAM-Laplace-Padé [2,2]** gives better results than **SAM** in terms of least squared error despite the t-value used as shown in Table 5-6. In general, the results validated the efficiency and the accuracy of the proposed two techniques: **SAM-Laplace** and **SAM-Laplace-Padé [2,2]** in terms of least square error. This due to the fact of combining **Laplace** and **Laplace-Padé [2,2]** methods into **SAM**. One point to mention here is that this conclusion is limited to solving this kind of strong nonlinear system of partial differential equations. Thus, for future work, we aim to apply these modification techniques to all types of equations.

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