Available online at sjuoz.uoz.edu.krd
journals.uoz.edu.krd
Science Journal of University of Zakho
Vol. 10, No. 2, pp. 43– 47, April-June 2022
p-ISSN: 2663-628X
e-ISSN: 2663-6298

SUMUDU-DECOMPOSITION METHOD TO SOLVE GENERALIZED HIROTA-SATSUMA COUPLED KDV SYSTEM

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Received: *11 Jan., 2021 /* ***Accepted:*** *20 Mar., 2022 /* ***Published:*** *18 Apr., 2022* ***https://doi.org/10.25271/sjuoz.2022.10.2.879***

ABSTRACT:

In this work, we take Adomian Decomposition Method (ADM) and combine it with Sumudu Transform method (STM). This connection between the two methods is called Sumudu-Decomposition Method (SDM), then use it to solve generalized Hirota-Satsuma Coupled kdv (H-SC kdv) systems and also we applied the STM, to find the approximate solutions of system one. Then we compare the approximate solutions of the two way with exact solitary solutions. Clarifying the best way through tables and drawings, then discussing the reason for the changes taking place in the roads and which one is closest to the exact solution.

KEYWORDS: Adomian Decomposition Method, Hirota-Satsuma coupled kdv systems, Sumudu Transform method.

1. INTRODUCTION

The coupled Kortweg-de Vries (Ckdv) equation describes the interaction of two long waves with different scattering relationships. There are several physical relationships in the CKdv equation. The evolution of one-dimensional long waves in a number of physical contexts, including ion plasma sound waves, shallow water waves with mild nonlinear restoring forces, long internal waves in a thickly layered ocean, and sound waves on a crystal lattice, is roughly explained. (Lawal, O. W., Loyimi, A.C. and Erinle-Ibrahim 2018).

We look at a generalized Hirota-Satsuma linked KdV equation, which was one of alfirst .'s equations introduced by (Wu et al. 1999). They presented a 4×4 matrix spectrum problem.Three potentials were proposed, along with a corresponding hierarchy of potentials.nonlinear equations; one of the most common hierarchical equations is (H-SC kdv system.in (Raslan 2004) he solved the system using decomposition method. Where (Lawal, O. W., Loyimi, A.C. and Erinle-Ibrahim 2018) solved it used Homotopy Perturbation Transform Method (HPTM).And (Yagmurlu, Karaagac, and Esen 2019), apply Lumped Galerkin finite element method using quadratic B-splines to solved it.

The decomposition approach has been proven (G. Adomian 1988; George Adomian 1994)to solve a vast class of linear and nonlinear, ordinary or partial, deterministic or stochastic differential equations effectively, easily, and accurately with approximates that converge quickly to accurate solutions.

In this paper, the Sumudu Transform method (Fethi Bin Muhammad Belgacem 2009; Fethi Bin Muhammed Belgacem and Karaballi 2006; Fethi Bin Muhammad Belgacem 2010), and Sumudu Decomposition Method (Bildik and Deniz 2016; Eltayeb and Kiliçman 2012; Eltayeb, Klçman, and Mesloub 2014; Ahmed and Elzaki 2015), are applied to the H-SC kdv systems.

The model based on mathematics of (H-SC kdv) equation are:

$$U_\tau = \frac{1}{2}U_{\chi\chi\chi} - 3UU_\chi + 3(VW)_\chi$$

$$V_\tau = -V_{\chi\chi\chi} + 3UV_\chi \qquad (1)$$

$$W_\tau = -W_{\chi\chi\chi} + 3UW_\chi$$

The exact solitary solution of Eq. (1) as in (Mehdi Hosseini, Mohyud-Din, and Ghaneai 2012)

are:

$$U(\chi,\tau) = \frac{1}{3}(\gamma - 2a^2) + 2a^2\tanh^2\big(a(\chi+\gamma\tau)\big)$$

$$V(\chi,\tau) = \frac{-4a^2c_0(\gamma+a^2)}{3c_1^2} + \frac{4a^2(\gamma+a^2)}{3c_1}\tanh\big(a(\chi+\gamma\tau)\big) \qquad (2)$$

$$W(\chi,\tau) = c_0 + c_1\tanh\big(a(\chi+\gamma\tau)\big)$$

And initial conditions are:

$$U(\chi,0) = \frac{1}{3}(\gamma - 2a^2) + 2a^2\tanh^2(a\chi)$$

$$V(\chi,0) = \frac{-4a^2c_0(\gamma+a^2)}{3c_1^2} + \frac{4a^2(\gamma+a^2)}{3c_1}\tanh(a\chi)$$

$$W(\chi,0) = c_0 + c_1\tanh(a\chi) \qquad (3)$$

Where $a, c_0, c_1 \neq 0$ and γ arbitrary constants.

2. BASIC IDEA OF STM

To comprehend the essence of **STM**, consider the inhomogeneous nonlinear partial differential equations of the following form with the following initial condition (Fethi Bin Muhammad Belgacem 2009; Fethi Bin Muhammed Belgacem and Karaballi 2006; Fethi Bin Muhammad Belgacem 2010):

$$\ell\omega(\chi,\tau) + G\omega(\chi,\tau) + N\omega(\chi,\tau) = h(\chi,\tau)$$

$$\omega(\chi,\tau) = \alpha \qquad (4)$$

if ℓ is first order derivative, G is a linear differential factor, NY represents the non-linear term and h(t) is the phrase that came from the source.

Taking the STM (denoted in the paper by S) on the equation (4), we have

$$S[\ell\omega(\chi,\tau)] + S[G\omega(\chi,\tau)] + S[N\omega(\chi,\tau)] = S[h(\chi,\tau)]$$

$$\omega(\chi,0) = \alpha \qquad (5)$$

using the ST differentiation with the initial circumstances listed above, shows

$$\frac{S[\ell\omega(\chi,\tau)] - \omega(\chi,0)}{s} + S[G\omega(\chi,\tau)] + S[N\omega(\chi,\tau)] = S[h(\chi,\tau)]$$

(6)

on both sides of the equation (6), use the Inverse STM, let's get

$$\omega(\chi,\tau) = H(\chi,\tau) - S^{-1}\big[sS[G\omega(\chi,\tau) + N\omega(\chi,\tau)]\big] \qquad (7)$$

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where $H(\chi,\tau)$ is a term that arises from the original term as well as the pre-specified beginning conditions.
As a result, the solution can be represented as an infinite series:

$$\omega(\chi,\tau)=\sum_{i=0}^{\infty}\omega_i(\chi,\tau) \quad (8)$$

$$\sum_{i=0}^{\infty}\omega_i(\chi,\tau)=H(\chi,\tau)-S^{-1}\left[sS\left[G\sum_{i=0}^{\infty}\omega_i(\chi,\tau)+N\sum_{i=0}^{\infty}\omega_i(\chi,\tau)\right]\right] \quad (9)$$

using standard STM and comparing both sides of the last equation, given by:

$$\omega_0(\chi,\tau)=H(\chi,\tau)$$
$$\omega_1(\chi,\tau)=-S^{-1}[sS[G\omega_0(\chi,\tau)+N\omega_0(\chi,\tau)]] \quad (10)$$
$$\omega_2(\chi,\tau)=-S^{-1}[sS[G\omega_1(\chi,\tau)+N\omega_1(\chi,\tau)]]$$

and the general relation is given by

$$\omega_{i+1}(\chi,\tau)=-S^{-1}[sS[G\omega_i(\chi,\tau)+N\omega_i(\chi,\tau)]],i\geq 0 \quad (11)$$

eventually, taking the STM of the right hand side of the final equation and then taking the Inverse STM.

3. BASIC IDEA OF SDM

In equation (9), when we find the nonlinear term by using Adomian polynomials (Bildik and Deniz 2016; Eltayeb and Kiliçman 2012; Eltayeb, Klçman, and Mesloub 2014; Ahmed and Elzaki 2015), the non-linear term can be broken down as

$$N\omega(\chi,\tau)=\sum_{i=0}^{\infty}C_i(\chi,\tau) \quad (12)$$

where C_i are Adomian polynomials of $\omega_0,\omega_1,\omega_2$, ... and we can calculate it by:

$$C_i=\frac{1}{i!}\frac{d^i}{d\hbar^i}\left[Z\left(\sum_{n=0}^{\infty}\hbar^n\omega_n\right)\right]_{\hbar=0}, i=0,1,2, \quad (13)$$

Substituting (8) and (12) into (7) , we get

$$\sum_{i=0}^{\infty}\omega_i(\chi,\tau)=H(\chi,\tau)-S^{-1}\left[sS\left[G\sum_{i=0}^{\infty}\omega_i(\chi,\tau)+\sum_{i=0}^{\infty}C_i(\chi,\tau)\right]\right] \quad (14)$$

on comparing both sides of last equation and by using standard ADM to find:

$$\omega_0(\chi,\tau)=H(\chi,\tau)$$
$$\omega_1(\chi,\tau)=-S^{-1}[sS[G\omega_0(\chi,\tau)+C_0]] \quad (15)$$
$$\omega_2(\chi,\tau)=-S^{-1}[sS[G\omega_1(\chi,\tau)+C_1]]$$

and the general relation is given by

$$\omega_{i+1}(\chi,\tau)=-S^{-1}[sS[G\omega_i(\chi,\tau)+C_i]],i\geq 0 \quad (16)$$

Eq.(16) called Sumudu-Decomposition Method (SDM)

4. APPLICATION

Analysis of numerical approaches is presented in this section such as **STM**, and **SDM** by applying them to the system of nonlinear partial differential Equations (1).
All numerical results were obtained using *MATHEMATICA* software utilizing all of the above approaches. This is owing to its ease of use and ability to manipulate data.
The numerical results of System (1) obtained by **STM** using three approximate terms can be seen below:

$$U(\chi,\tau)=\frac{1}{3}(\gamma-2a^2)+4\gamma a^3\tau\text{Sech}[a\chi]^2\text{Tanh}[a\chi]+2a^2\text{Tanh}[a\chi]^2-\frac{1}{6}\gamma a^5\text{Sech}[a\chi]^6(120a\tau^2-75a\tau^2\text{Cosh}[3a\chi]\text{Sech}[a\chi]+3a\tau^2\text{Cosh}[5a\chi]\text{Sech}[a\chi]+8\gamma^2\tau^3\text{Sech}[a\chi]\text{Sinh}[3a\chi]-40\gamma a^2\tau^3\text{Sech}[a\chi]\text{Sinh}[3a\chi]+8\gamma^2\tau^3\text{Tanh}[a\chi]+248\gamma a^2\tau^3\text{Tanh}[a\chi])$$

$$V(\chi,\tau)=-\frac{4c_0a^2(\gamma+a^2)}{3{c_1}^2}+\frac{4\gamma a^3(\gamma+a^2)\tau\text{Sech}[a\chi]^2}{3c_1}+\frac{4a^2(\gamma+a^2)\text{Tanh}[a\chi]}{3c_1}-\frac{4\gamma a^6(\gamma+a^2)\text{Sech}[a\chi]^4\text{Tanh}[a\chi]}{3c_1}\left((9\tau^2-t^2\text{Cosh}[3a\chi]\text{Sech}[a\chi]+8\gamma a\tau^3\text{Tanh}[a\chi])\right)$$

$$W(\chi,\tau)=c_0+\gamma c_1a\tau\text{Sech}[a\chi]^2+c_1\text{Tanh}[a\chi]-\gamma c_1a^4\text{Sech}[a\chi]^4\text{Tanh}[a\chi](9\tau^2-\tau^2\text{Cosh}[3a\chi]\text{Sech}[a\chi]+8\gamma a\tau^3\text{Tanh}[a\chi])$$

The numerical results of System (1) obtained by **SDM** using three approximate terms can be seen below:

$$U(\chi,\tau)=\frac{1}{3}(\gamma-2a^2)-2\gamma^2a^4\tau^2(-2+\text{Cosh}[2a\chi])\text{Sech}[a\chi]^4+4\gamma a^3\tau\text{Sech}[a\chi]^2\text{Tanh}[a\chi]+2a^2\text{Tanh}[a\chi]^2$$

$$V(\chi,\tau)=-\frac{4c_0a^2(\gamma+a^2)}{3{c_1}^2}+\frac{4\gamma a^3(\gamma+a^2)\tau\text{Sech}[a\chi]^2}{3c_1}+\frac{4a^2(\gamma+a^2)\text{Tanh}[a\chi]}{3c_1}-\frac{4\gamma^2a^4(\gamma+a^2)\tau^2\text{Sech}[a\chi]^2\text{Tanh}[a\chi]}{3c_1}$$

$$W(\chi,\tau)=c_0+\gamma c_1a\tau\text{Sech}[a\chi]^2+c_1\text{Tanh}[a\chi]-\gamma^2c_1a^2\tau^2\text{Sech}[a\chi]^2\text{Tanh}[a\chi]$$

Tables 1-3 show the differences between **Exact** and approximate solutions using **STM** and **SDM** for $U(\chi,\tau),V(\chi,\tau)$ and $W(\chi,\tau)$, respectively. From the tables, we notice that the solution by **SDM** is more convergent than **STM** way, when χ = 1 [arbitrary chosen], $a=0.1,c_0=1.5,c_1=0.\gamma=1.5$ and $\tau\in[0,1]$.

Table 1

$U(\chi,\tau)$	*Exact*	*STM*	*SDM*
$(1,0)$	0.4935320075	0.4935320075	0.4935320075
$(1,0.1)$	0.4935955187	0.4935910862	0.4935955368
$(1,0.2)$	0.4936675614	0.4936498365	0.4936677111
$(1,0.3)$	0.4937480088	0.4937081502	0.4937485305
$(1,0.4)$	0.4938367203	0.493765919	0.4938379949
$(1,0.5)$	0.4939335417	0.4938230347	0.4939361044
$(1,0.6)$	0.4940383061	0.4938793889	0.4940428589
$(1,0.7)$	0.4941508341	0.4939348735	0.4941582585
$(1,0.8)$	0.4942709348	0.4939893801	0.4942823032
$(1,0.9)$	0.4943984062	0.4940428006	0.4944149929
$(1,1)$	0.4945330364	0.4940950266	0.4945563276

Table 2

$V(\chi,\tau)$	*Exact*	*STM*	*SDM*
$(1,0)$	−2.99993351	−2.99993351	−2.99993351
$(1,0.1)$	−2.996948197	−2.996943745	−2.99694798
$(1,0.2)$	−2.99397312	−2.993954452	−2.99397139
$(1,0.3)$	−2.991009557	−2.990965633	−2.991003741
$(1,0.4)$	−2.988058768	−2.98797729	−2.988045031
$(1,0.5)$	−2.985121988	−2.984989425	−2.985095262
$(1,0.6)$	−2.982200431	−2.98200204	−2.982154433
$(1,0.7)$	−2.979295281	−2.979015138	−2.979222544
$(1,0.8)$	−2.976407697	−2.97602872	−2.976299596
$(1,0.9)$	−2.973538807	−2.973042789	−2.973385588
$(1,1)$	−2.970689709	−2.970057347	−2.970480519

Table 3

$W(\chi,\tau)$	*Exact*	*STM*	*SDM*
$(1,0)$	1.509966799	1.509966799	1.509966799
$(1,0.1)$	1.511449571	1.511451782	1.511449679
$(1,0.2)$	1.512927258	1.51293653	1.512928117
$(1,0.3)$	1.514399227	1.514421043	1.514402115
$(1,0.4)$	1.51586485	1.51590532	1.515871673
$(1,0.5)$	1.517323516	1.517389359	1.51733679
$(1,0.6)$	1.518774621	1.518873159	1.518797467
$(1,0.7)$	1.520217576	1.52035672	1.520253703

$(\mathbf{1},\mathbf{0.8})$	1.521651806	1.52184004	1.521705499
$(\mathbf{1},\mathbf{0.9})$	1.523076751	1.523323118	1.523152854
$(\mathbf{1},\mathbf{1})$	1.524491866	1.524805954	1.524595769

and, Tables 4-6 show the difference in absolute error between the exact and approximate solutions by **STM and SDM** for $U(\chi,\tau), V(\chi,\tau)$ and $W(\chi,\tau)$ respectively. From these tables, there is a clear change in the results that shows the accuracy of the solution by **SDM**, when $\chi = 1, a = 0.1, c_0 = 1.5, c_1 = 0. \gamma = 1.5$and $\tau \in [\,0{,}1\,]$.

Table 4

$(\boldsymbol{\chi},\boldsymbol{\tau})$	$\lvert \boldsymbol{USTM-UExact} \rvert$	$\lvert \boldsymbol{USDM-UExact} \rvert$
$(\mathbf{1},\mathbf{0})$	0	0
$(\mathbf{1},\mathbf{0.1})$	4.4325×10^{-6}	1.8113×10^{-8}
$(\mathbf{1},\mathbf{0.2})$	1.77249×10^{-5}	1.49773×10^{-7}
$(\mathbf{1},\mathbf{0.3})$	3.98586×10^{-5}	5.21718×10^{-7}
$(\mathbf{1},\mathbf{0.4})$	7.08013×10^{-5}	1.27464×10^{-6}
$(\mathbf{1},\mathbf{0.5})$	1.10507×10^{-4}	2.56269×10^{-6}
$(\mathbf{1},\mathbf{0.6})$	1.58917×10^{-4}	4.55286×10^{-6}
$(\mathbf{1},\mathbf{0.7})$	2.15961×10^{-4}	7.42446×10^{-6}
$(\mathbf{1},\mathbf{0.8})$	2.81555×10^{-4}	1.13684×10^{-5}
$(\mathbf{1},\mathbf{0.9})$	3.55606×10^{-4}	1.65866×10^{-5}
$(\mathbf{1},\mathbf{1})$	4.3801×10^{-4}	2.32912×10^{-5}

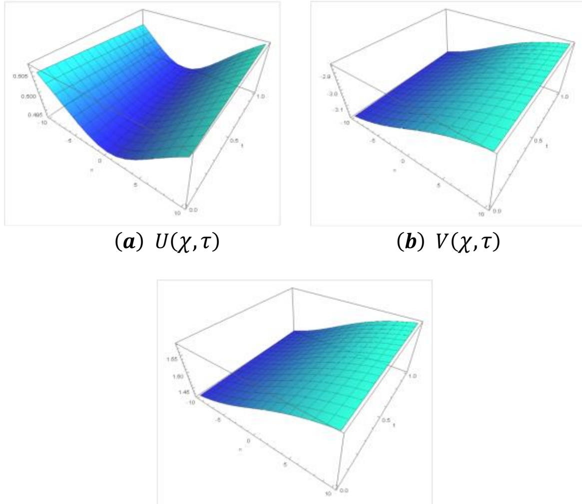
Table 5

$(\boldsymbol{\chi},\boldsymbol{\tau})$	$\lvert \boldsymbol{VSTM-VExact} \rvert$	$\lvert \boldsymbol{VSDM-VExact} \rvert$
$(\mathbf{1},\mathbf{0})$	0	0
$(\mathbf{1},\mathbf{0.1})$	4.45179×10^{-6}	2.16888×10^{-7}
$(\mathbf{1},\mathbf{0.2})$	1.86676×10^{-5}	1.72938×10^{-6}
$(\mathbf{1},\mathbf{0.3})$	4.39241×10^{-5}	5.81633×10^{-6}
$(\mathbf{1},\mathbf{0.4})$	8.14779×10^{-5}	1.37364×10^{-5}
$(\mathbf{1},\mathbf{0.5})$	1.32563×10^{-4}	2.67261×10^{-5}
$(\mathbf{1},\mathbf{0.6})$	1.9839×10^{-4}	4.59974×10^{-5}
$(\mathbf{1},\mathbf{0.7})$	2.80143×10^{-4}	7.27364×10^{-5}
$(\mathbf{1},\mathbf{0.8})$	3.78977×10^{-4}	1.08101×10^{-4}
$(\mathbf{1},\mathbf{0.9})$	4.96018×10^{-4}	1.5322×10^{-4}
$(\mathbf{1},\mathbf{1})$	6.32363×10^{-4}	2.0919×10^{-4}

Table 6

$(\boldsymbol{\chi},\boldsymbol{\tau})$	$\lvert \boldsymbol{WSTM-WExact} \rvert$	$\lvert \boldsymbol{WSDM-WExact} \rvert$
$(\mathbf{1},\mathbf{0})$	0	0
$(\mathbf{1},\mathbf{0.1})$	2.21115×10^{-6}	1.07726×10^{-7}
$(\mathbf{1},\mathbf{0.2})$	9.27198×10^{-6}	8.58961×10^{-7}
$(\mathbf{1},\mathbf{0.3})$	2.18166×10^{-5}	2.88891×10^{-6}
$(\mathbf{1},\mathbf{0.4})$	4.04692×10^{-5}	6.82273×10^{-6}
$(\mathbf{1},\mathbf{0.5})$	6.58427×10^{-5}	1.32745×10^{-5}
$(\mathbf{1},\mathbf{0.6})$	9.85383×10^{-5}	2.28464×10^{-5}
$(\mathbf{1},\mathbf{0.7})$	1.39144×10^{-4}	3.61273×10^{-5}
$(\mathbf{1},\mathbf{0.8})$	1.88234×10^{-4}	5.36926×10^{-5}
$(\mathbf{1},\mathbf{0.9})$	2.46367×10^{-4}	7.61026×10^{-5}
$(\mathbf{1},\mathbf{1})$	3.14087×10^{-4}	1.03902×10^{-4}

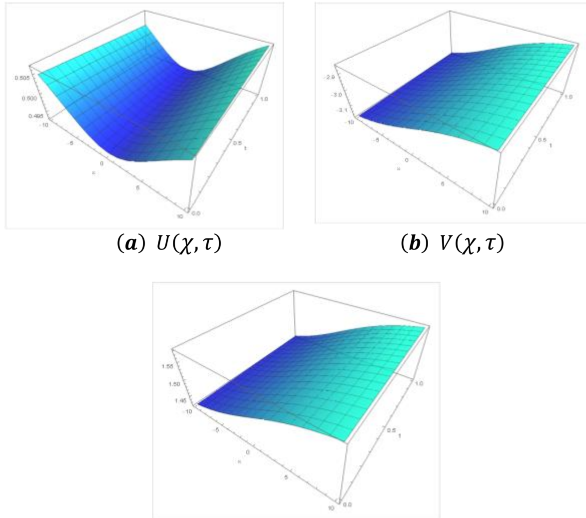
Also, the **Figure**1-3, below are the surfaces for the solitary solution of **H-SC kdv** system, **STM,** and **SDM** respectively, when $\chi \in [-10,\,10]$ and $\tau \in [0{,}1]$.



$(\boldsymbol{a})\ U(\chi,\tau)$ $(\boldsymbol{b})\ V(\chi,\tau)$

$(\boldsymbol{c})\ W(\chi,\tau)$

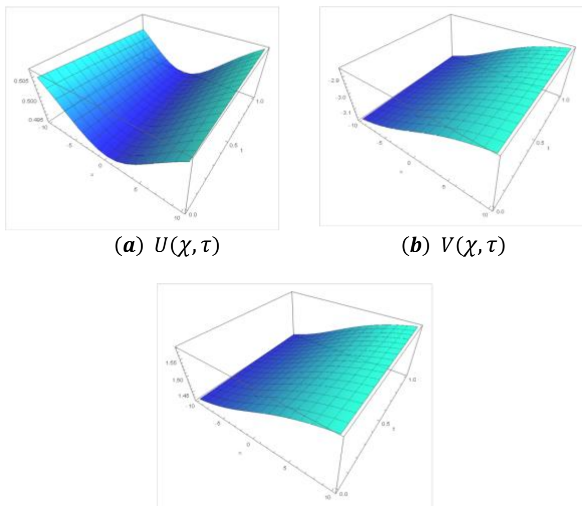
Figure 1 Surfaces of **Exact** solutions.



$(\boldsymbol{a})\ U(\chi,\tau)$ $(\boldsymbol{b})\ V(\chi,\tau)$

$(\boldsymbol{c})\ W(\chi,\tau)$

Figure 2 Surfaces of **STM** solutions.

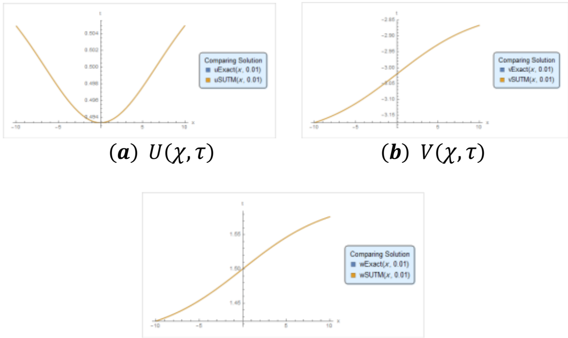


$(\boldsymbol{a})\ U(\chi,\tau)$ $(\boldsymbol{b})\ V(\chi,\tau)$

$(\boldsymbol{c})\ W(\chi,\tau)$

Figure 3 Surfaces of **SDM** solutions.

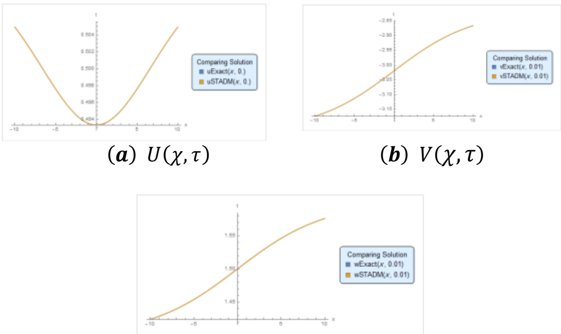
The curves in **Figure**4, and **Figure**5, show that how the **STM, SDM** curves are close to the solitary solution curve, when $\chi \in [-10,\,10]$ and $\tau \in [0{,}10]$, τ =0.01.



($\boldsymbol{a}$) $U(\chi,\tau)$ ($\boldsymbol{b}$) $V(\chi,\tau)$

($\boldsymbol{c}$) $W(\chi,\tau)$

Figure 4 Curves of **STM** for $\chi\epsilon[-10{,}10], \tau\epsilon[0{,}10]$

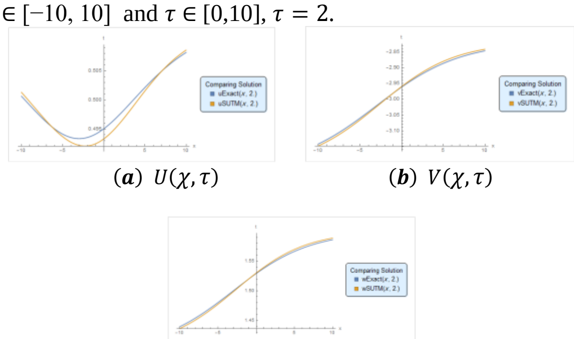


($\boldsymbol{a}$) $U(\chi,\tau)$ ($\boldsymbol{b}$) $V(\chi,\tau)$

($\boldsymbol{c}$) $W(\chi,\tau)$

Figure 5 Curves of **SDM** for $\chi\epsilon[-10{,}10], \tau\epsilon[0{,}10]$

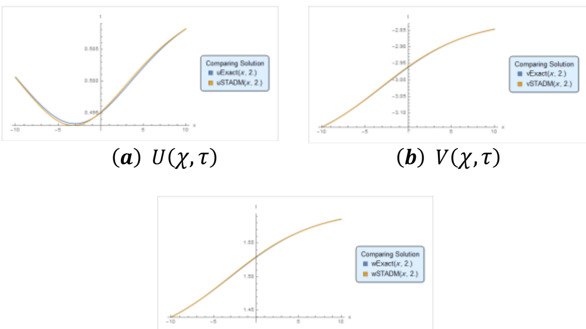
The curves in **Figure**6, and **Figure**7, show that how the **STM, SDM** curves are close to the solitary solution curve, when $\chi \in [-10,\ 10]$ and $\tau \in [0{,}10]$, $\tau = 2$.



($\boldsymbol{a}$) $U(\chi,\tau)$ ($\boldsymbol{b}$) $V(\chi,\tau)$

($\boldsymbol{c}$) $W(\chi,\tau)$

Figure 6 Curves of **STM** for $\chi\epsilon[-10{,}10], \tau\epsilon[0{,}10]$



($\boldsymbol{a}$) $U(\chi,\tau)$ ($\boldsymbol{b}$) $V(\chi,\tau)$

($\boldsymbol{c}$) $W(\chi,\tau)$

Figure 7 Curves of **SDM** for $\chi\epsilon[-10{,}10], \tau\epsilon[0{,}10]$

Finally, as for the graphics, we note how the best method of convergence to the solitary solution is the **SDM** than **STM** through clarification when we taking the $\tau = 2$.

Now, Tables 7,8 and 9 show the least square errors between **STM** and **SDM** respectively, from this table that is clear the change and best method when we decomposed the method using Adomian Decomposition method , when $\chi = 1, a = 0.1, c_0 = 1.5, c_1 = 0. \gamma = 1.5$ and $\tau \in [\,0{,}1\,]$.

Table 7

$U(\chi,\tau)$	$\lvert USTM - UExact\rvert$	$\lvert USDM - UExact\rvert$
Least square error	4.88622×10^{-8}	1.03118×10^{-10}

Table 8

$V(\chi,\tau)$	$\lvert VSTM - VExact\rvert$	$\lvert VSDM - VExact\rvert$
Least square error	9.33888×10^{-8}	8.72688×10^{-9}

Table 9

$W(\chi,\tau)$	$\lvert WSTM - WExact\rvert$	$\lvert WSDM - WExact\rvert$
Least square error	2.3039×10^{-8}	2.15292×10^{-9}

5. CONCLUSION

SDM and **STM** are two method which it has been applied of non-linear **H-SC KdV** equations. We used one example of the equation to compare our solutions to the precise solution, and we demonstrated that both methods are extremely accurate and successful in solving the problem (**H-SC KdV**) equation. However, it is clear from the Table 4, Table 5 and Table 6 that the difference in absolute error between exact and approximate solution by **SDM** is better than the approximate solutions **STM**, This means that when we decomposed the method using Adomian Decomposition method we get a better result. Also from the curves in Figures 4-7 The convergence of the solution indicates which one is closer**.**

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