

## OPTIMUM TILT ANGLE FOR SOUTH FACING FLAT SOLAR COLLECTORS IN DUHOK CITY

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### Abstract

One of the important parameters that affect the performance of a solar collector is its' tilt angle with the horizon. This is because of the variation of tilt angle changes the amount of solar radiation reaching the collector surface. In this paper, the global irradiation observed in Duhok from 1990 to 2007 was used to estimate the optimal tilt angle for solar collectors. The observed data are resolved into diffusion, beam and albedo components. Total solar radiation on the solar collector surface with an optimum tilt angle is computed for specific periods. It is found that the optimum tilt angle changes between  $0^\circ$  (June) and  $62^\circ$  (December) throughout the year. In winter (December, January, and February) the tilt should be  $58^\circ$ , in spring (March, April, and May)  $20^\circ$ , in summer (June, July, and August)  $5^\circ$ , and in autumn (September, October, and November)  $46.3^\circ$ . The yearly average of this value was found to be  $32.33^\circ$  and this would be the optimum fixed tilt throughout the year.

*Keywords:* Solar radiation, Collector angle, Duhok city Kurdistan-Iraq

### Nomenclature

$H_0$ : monthly average daily extraterrestrial radiation on a horizontal surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$I$ : solar constant ( $\text{W/m}^2$ )

$H$ : monthly average daily global radiation on a horizontal surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$H_b$ : monthly average daily direct radiation on a horizontal surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$H_d$ : monthly average daily diffuse radiation on a horizontal surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$H_t(\beta)$ : monthly average daily total radiation on a tilted surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$H_b(\beta)$ : monthly average daily direct radiation on a tilted surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$H_d(\beta)$ : monthly average daily diffuse radiation on a tilted surface ( $\text{kW.h/m}^2 \cdot \text{day}$ )

$K_t$ : the monthly-average clearness index

$N$ : number of the day of year starting from the first January

$R_b$ : the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month

$\beta$ : tilt of the surface from horizontal ( $^\circ$ )

$\delta$ : solar declination angle ( $^\circ$ )

$\varnothing$ : latitude of site ( $^\circ$ )

$\rho$ : ground reflectance ( $\approx 0.2$ )

$\omega_s$ : sunshine hour angle for the month ( $^\circ$ )

$\omega_{ss}$ : sunset hour angle for the tilted surface ( $^\circ$ )

### Introduction

Solar radiation on tilted planes is very important to engineers designing plate collectors, photovoltaic systems and other solar energy collecting devices (Salem Nijmeh et al, 2000). Solar radiation data is usually measured in the form of global solar radiation on a horizontal surface at the latitude of interest. Flat-plate solar collectors are tilted so that they capture maximum radiation and the problem of calculating solar radiation on tilt surface is in determining the relative amount of beam and diffuse containing in measured global solar radiation on horizontal surface. Since the solar collectors are positioned at angle it is necessary to calculate the optimal angle which maximizes the amount of collected energy (D. Ibrahim, 1995). It is generally known that in the northern hemisphere the optimum collector orientation is south facing (e.i.  $\gamma = 0$ ) and the optimum tilt depends upon the latitude, local climate and the solar declination (Koray, 2006). Researchers like

Duffie and Beckman suggested that the tilt angle should be  $10^{\circ}$ – $15^{\circ}$  more than the latitude during winter and  $10^{\circ}$ – $15^{\circ}$  less than the latitude during summer (Duffie et al, 1991)

Hamdy K. Elminir concluded that the tilt angle for Helwan, Egypt is  $(\theta \pm 15)^{\circ}$  (Hamdy K. Elminir et al, 2005). Can Ertekin in a study on 158 cities in Turkey found that the optimum tilt angle to be  $(\theta - 8.14)^{\circ}$  in winter months and  $(\theta - 35.17)^{\circ}$  for summer months (Can Ertekin et al, 2008). Tian Pau Chang propose that the yearly optimum tilt angle is  $(0.764\theta + 2.14)^{\circ}$  for  $(\theta \leq 65)^{\circ}$  and  $(0.224\theta + 33.65)^{\circ}$  otherwise (Tian Pau, 2008)

The objective of this study is to determine the optimum tilt angle for a fixed south facing solar collector and the total solar radiation on a tilted surface in Duhok, Kurdistan Region of Iraq. Total solar radiation on the solar collector surface with an optimum tilt angle is computed for specific periods (monthly, seasonal, and yearly).

### Theoretical Analysis

Monthly average daily total solar radiation  $H(\beta)$  on a tilted surface at angle  $(\beta)$ , is normally estimated by individually considering the direct beam  $H_b(\beta)$ , diffuse  $H_d(\beta)$  and reflected components  $H_r(\beta)$  of the radiation on a tilted surface. Thus for a surface tilted at a slope angle from the horizontal, the incident total radiation is given by (Luque et al, 2003):

$$H_t(\beta) = H_b(\beta) + H_d(\beta) + H_r(\beta) \dots\dots\dots (1)$$

The daily beam solar radiation received on a horizontal surface may be expressed as (Luque et al, 2003):

$$H_b = (H - H_d) \dots\dots\dots (2)$$

And daily beam solar radiation on an inclined surface can be expressed as:

$$H_b(\beta) = (H - H_d)R_b \dots\dots\dots (3)$$

Where  $H$  and  $H_d$  are the monthly-average daily global and diffuse radiation on a horizontal surface respectively, and  $R_b$  is the ratio of the average beam radiation on the tilted surface to that on a horizontal surface for each month.  $R_b$  is a function of the transmittance of the atmosphere, which depends upon the atmospheric cloudiness, water vapor, and particulate concentration Liu and Jordan have suggested that  $R_b$  can be estimated to be the ratio of extraterrestrial radiation on the tilted surface to that on a horizontal surface for each month. For a surface facing directly towards the equator (Liu et al, 1960):

$$R_b = \frac{\cos(\theta - \beta)\cos\delta\sin\omega_s + \left(\frac{\pi}{180}\right)\omega_s\sin(\theta - \beta)\sin\delta}{\cos\theta\cos\delta\sin\omega_s + \left(\frac{\pi}{180}\right)\omega_s\sin\theta\sin\delta} \dots\dots\dots (4)$$

Where  $\beta$  is the tilt of the surface from horizontal,  $\theta$  is the latitude,  $\omega_s$  is the sunrise hour angle,  $\omega_{ss}$  is the sunset hour angle for the tilted surface and  $\delta$  is the solar declination angle,  $\omega_s$ ,  $\omega_{ss}$  and  $\delta$  can be found by using the following equations (Liu et al, 1960. Gilbert M. Masters, 2004. Duffie et al 1991):

$$\omega_s = \arccos(-\tan\delta\tan\theta) \dots\dots\dots (5)$$

$$\omega_{ss} = \min[\omega_s = (\arccos(-\tan\theta\tan\delta)), \arccos(-\tan(\theta - \beta)\tan\delta)] \dots\dots\dots (6)$$

$$\delta = 23.45 \sin\left(\frac{360(N+284)}{365}\right) \dots\dots\dots (7)$$

Where N is the number of the day of the year starting from the first of January

In order to determine monthly average daily diffuse solar radiation on a horizontal surface the following correlation was used (Page, 1964):

$$H_d = (1 - 1.13K_t)H \dots\dots\dots (8)$$

For an inclined surface the diffuse solar radiation is given by (Luque et al, 2003):

$$H_d(\beta) = \frac{H_d}{2} (1 + \cos\beta) \dots\dots\dots (9)$$

The monthly average clearness index (Kt) is the ratio of monthly average daily radiation on a horizontal surface (H) (kW.h m<sup>-2</sup> d<sup>-1</sup>) to monthly average daily extraterrestrial radiation on a horizontal surface (Ho) (kW.h m<sup>-2</sup> d<sup>-1</sup>). Kt and Ho can be calculated from the following equation (K.N Liou, 1992):

$$K_t = \frac{H}{H_o} \dots\dots\dots (10)$$

$$H_o = I \times \frac{24}{\pi} \times (1 + 0.003 \cos(\frac{360N}{365})) \times (\frac{\pi}{180} \omega_s \sin\delta \sin\phi + \cos\delta \cos\phi \sin\omega_s). \dots\dots\dots (11)$$

Where I is the solar constant (1367 W m<sup>-2</sup>)

The daily ground-reflected radiation may be written as (Luque et al, 2003):

$$H_r(\beta) = \frac{H_p}{2} (1 - \cos\beta) \dots\dots\dots (12)$$

$\rho$  is the ground reflectance ( $\approx 0.2$ ).

Substituting equations (3), (9) and (12) in equation (1) one can get:

$$H_t(\beta) = (H - H_d)R_b + \frac{H_d}{2} (1 + \cos\beta) + \frac{H_p}{2} (1 - \cos\beta) \dots\dots\dots (13)$$

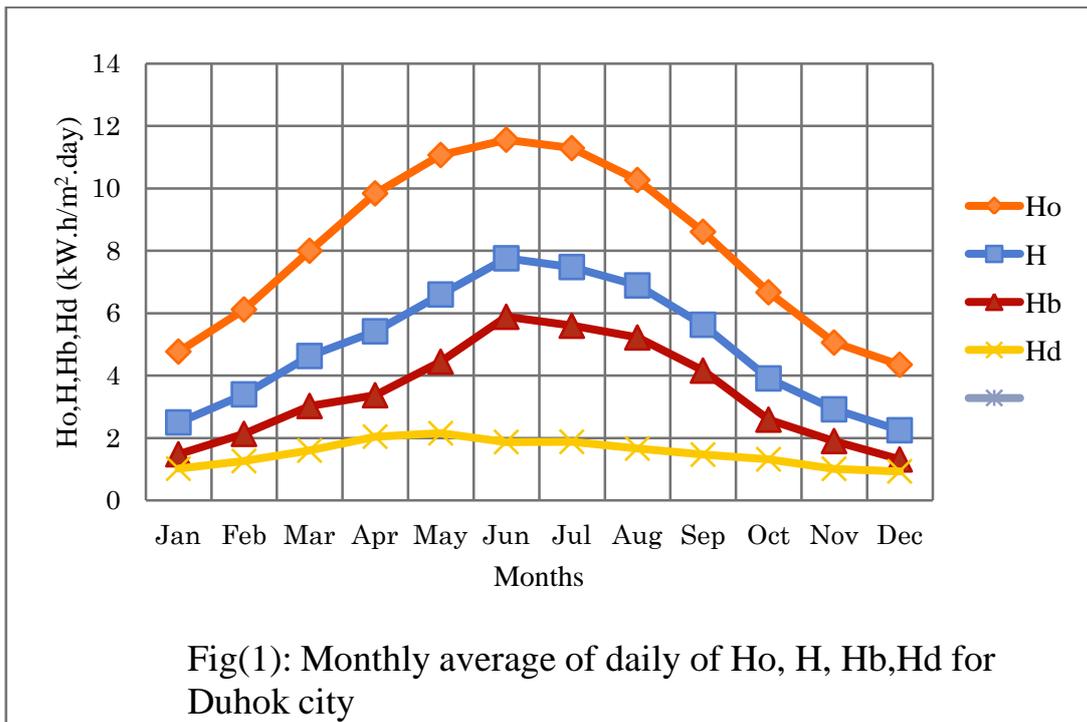
**Results and Discussion**

Since there is no ground base satation for measuring solar radiation data in Duhok city, the global solar radiation data was taken from the NASA surface meteorology and solar energy

satellites (NASA, 1997) and the values of Ho, Kt, Hb and Hd were found by using equations (11),(10),(2) and (8) respectively and the results are shown in table (1) and plotted in figure (1).

**Table (1):** Monthly average of daily of (Ho, H, Hb, Hd and Kt).

Month	Ho(kW.h/m <sup>2</sup> .day)	H(kW.h/m <sup>2</sup> .day)	Hb(kW.h/m <sup>2</sup> .day)	Hd(kW.h/m <sup>2</sup> .day)	Kt
Jan	4.77	2.5	1.480608	1.019392	0.524109
Feb	6.12	3.4	2.134444	1.265556	0.555556
Mar	8	4.63	3.027962	1.602038	0.57875
Apr	9.84	5.42	3.373509	2.046491	0.550813
May	11.07	6.6	4.446504	2.153496	0.596206
Jun	11.56	7.76	5.886322	1.873678	0.67128
Jul	11.29	7.48	5.599996	1.880004	0.662533
Aug	10.27	6.89	5.223318	1.666682	0.670886
Sep	8.61	5.63	4.159988	1.470012	0.653891
Oct	6.67	3.91	2.590038	1.319962	0.586207
Nov	5.06	2.92	1.904117	1.015883	0.577075
Dec	4.35	2.25	1.315086	0.934914	0.517241

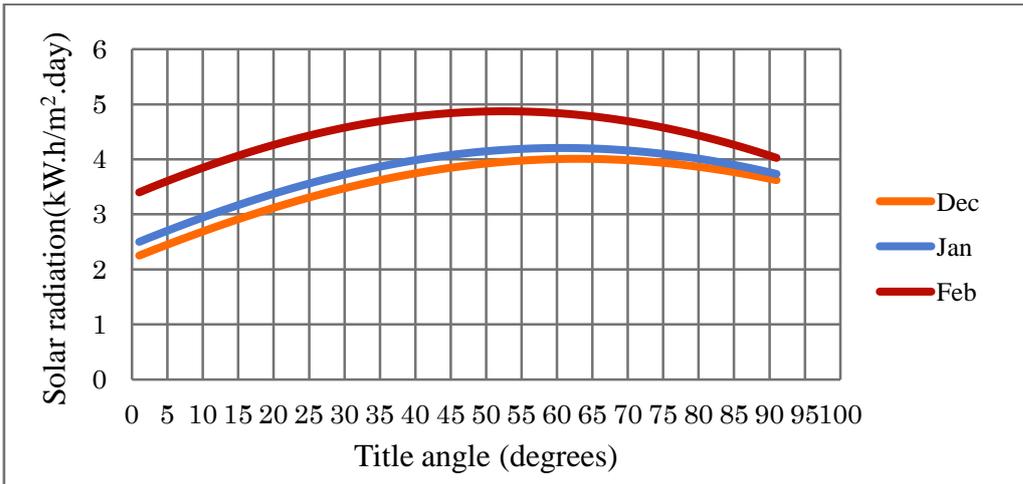


Equation (13) was used to calculate the monthly average daily total radiation on a south-facing surface. Figures (2), (3), (4) and (5) show the average daily total solar radiation on a south-facing surface when the angle of the tilt varied from 0° to 90° in steps of 1°. It can be seen that there is only one angle of tilt that offers

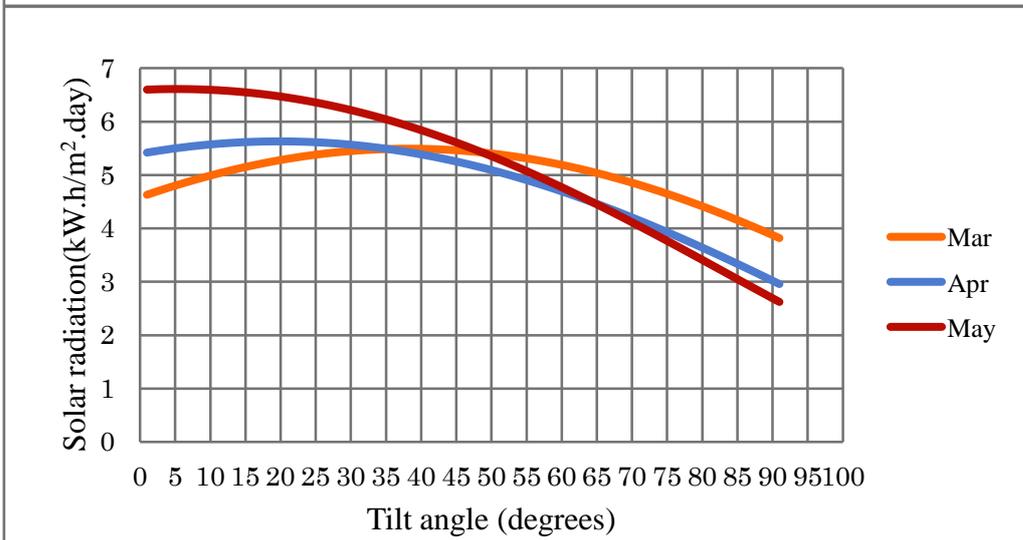
maximum solar radiation interception to south facing inclined surface of a flat plate collector. Optimum tilt angle and the monthly-average daily total solar radiation on an optimum tilted surface are given in Table 2. It is found that the optimum tilt angle was changing between 0° (June) and 62° (December) throughout a year.

**Table (2):** Optimum tilt angle ( $\beta_{opt}$ ) and monthly-average daily solar radiation on a tilted surface ( $H_{opt}$ ) for each month of the year for a south-facing solar collector in Duhok.

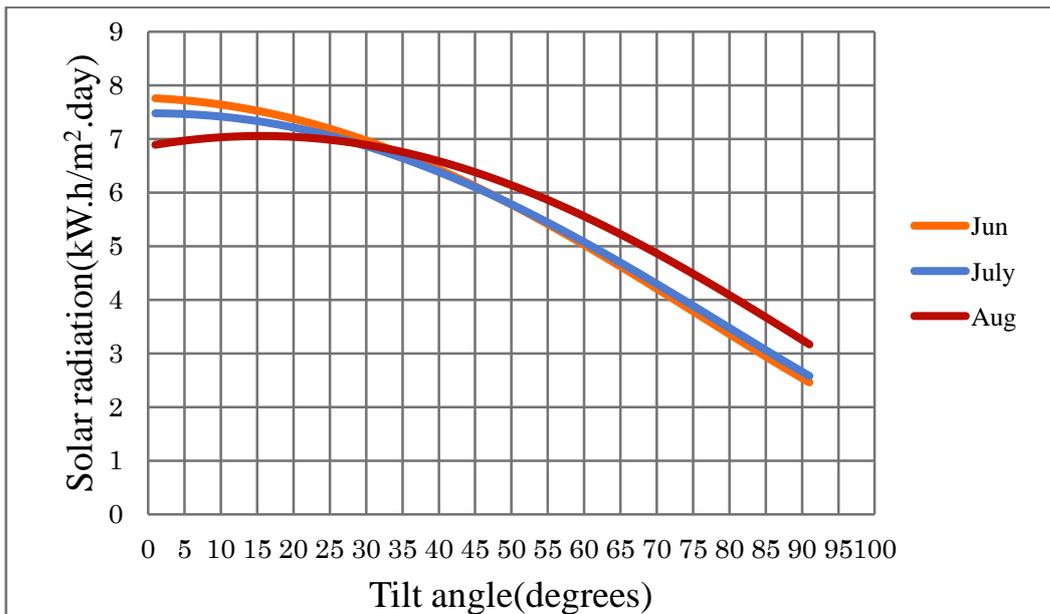
Months	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$\beta_{opt}$	60	52	37	19	4	0	0	15	32	48	59	62
$H_{opt}(Kw.h/m^2.day)$	4.203	4.873	5.490	5.630	6.61	7.76	7.48	7.053	6.422	5.312	4.911	4.009



Fig(2): Monthly average of daily solar radiation on tilted surfaces (Winter months)



Fig(3): Monthly average of daily solar radiation on tilted surfaces (Spring months)



Fig(4): Monthly average of daily solar radiation on tilted surfaces (Summer months)

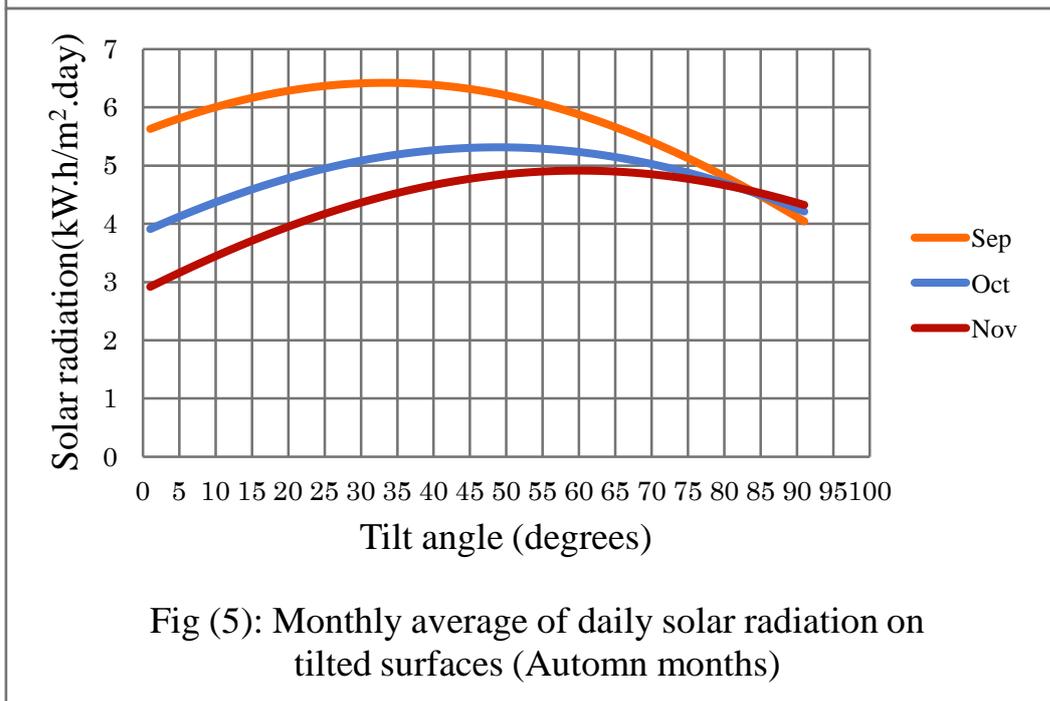


Fig (5): Monthly average of daily solar radiation on tilted surfaces (Autumn months)

Table 3 and Figure (6) show the tilt angle for each month of the year when the solar collector is tilted at the optimum angle, which clearly showed the seasonal average, and the yearly average. The seasonal average was calculated by finding the average value of the tilt angle for each season, and the implementation of this requires that the collector tilt be changed four times a year. In winter (December, January, and

February) the tilt should be 58° and the total solar radiation falling on the collector plane during this season is 390 (Kw.h/m<sup>2</sup>). In spring (March, April, and May), the optimal tilt angle is 20° with the total solar radiation of about 533.1 (Kw.h/m<sup>2</sup>) intercepted by the collector s' tilted at this angle .The optimum angle of tilt of flat collector in summer (June, July, and August) is 5°, and the total solar radiation is 678.9

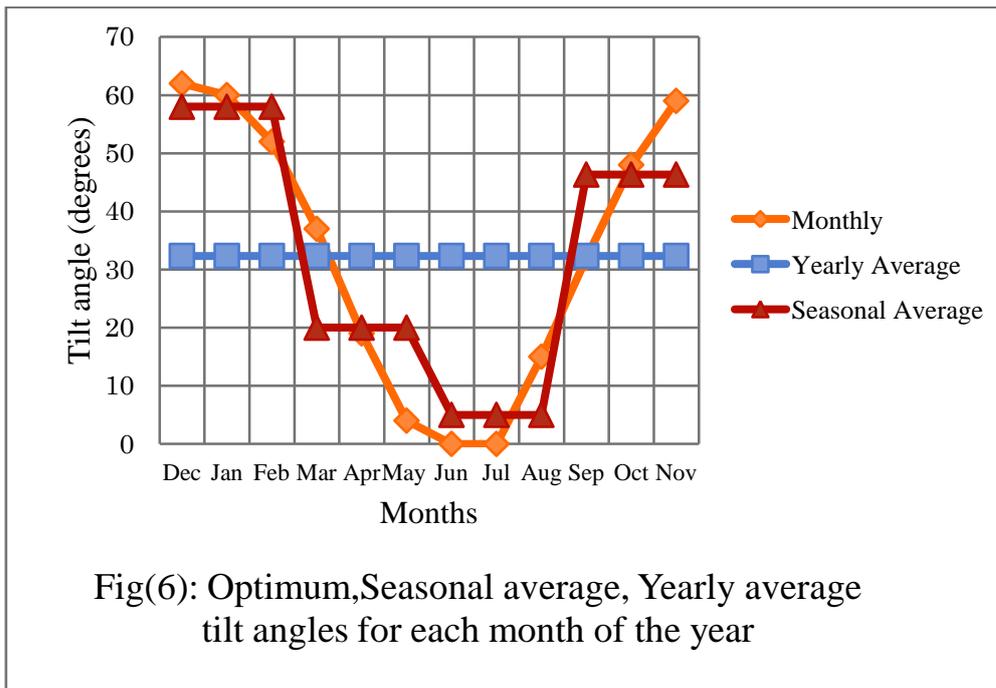
(Kw.h/m<sup>2</sup>). During autumn (September, October, and November) the optimal tilt angle is 46.3° and the total solar radiation incident on a collector tilted at this angle is 497 (Kw.h/m<sup>2</sup>). The yearly average tilt was calculated by finding the average value of the tilt angle for all months of the year. The yearly average of this value found to be 32.33°, the total solar radiation reaching the

collector plane tilted at this angle is 1994 (kW.h/m<sup>2</sup>). Figure (7) shows the monthly average solar energy collected when the angle of the tilt is optimum, and when the seasonal and yearly average angles are used.

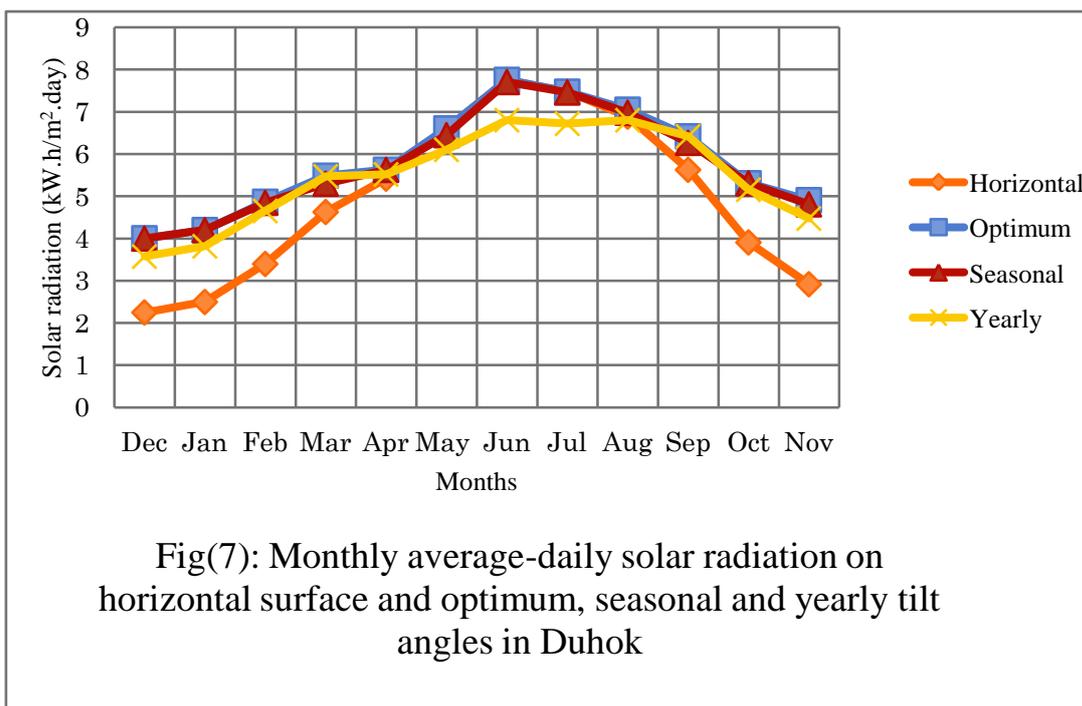
This amount of yearly solar radiation 1994 (kW.h/m<sup>2</sup>) is enough for heating water for average family in Kurdistan.

**Table (3):** Seasonal and yearly average tilt angle and monthly-average daily solar radiation on a tilted south facing surface in Duhok city

Month	Season	Seasonal average		H(kW.h/m <sup>2</sup> .season)	Yearly average
		$\beta^\circ$	H (Kw.h/m <sup>2</sup> .day)		
Dec	Winter	58	4.001	390	3.5749
Jan			4.202		3.819
Feb			4.848		4.656
Mar	Spring	20	5.301	533.1	5.475
Apr			5.629		5.520
May			6.450		6.104
Jun	Summer	5	7.708	678.9	6.807
Jul			7.460		6.722
Aug			6.982		6.804
Sep	Autumn	46.3	6.269	497	6.422
Oct			5.311		5.156
Nov			4.812		4.475
					Yearly Total 1994.365 (kW.h/m <sup>2</sup> )



Fig(6): Optimum, Seasonal average, Yearly average tilt angles for each month of the year



Fig(7): Monthly average-daily solar radiation on horizontal surface and optimum, seasonal and yearly tilt angles in Duhok

**Conclusions:**

In the light of the preceding results, the following conclusions can be drawn:

- 1- The results show that the average optimum tilt angle for the summer months is  $5^{\circ}$  ( $\bar{\theta} - 31.5^{\circ}$ ), for the winter months  $58^{\circ}$  ( $\bar{\theta} + 21.5^{\circ}$ ), for spring months  $20^{\circ}$  ( $\bar{\theta} - 16.5^{\circ}$ )

and for autumn months  $46.33^{\circ}$  ( $\bar{\theta} + 9.83^{\circ}$ ). With the yearly fixed angle  $32^{\circ}$  ( $\bar{\theta} - 4.5^{\circ}$ ).

2- Designing solar collector in which the tilt angle can be changed on monthly basis provides better efficiency.

3- When the solar collector is set on horizontal there is a reduction of 14.7% as compared to monthly, but when the solar collector is set at the seasonal tilt angle there is a reduction of

1.13% as compared to monthly, similarly there is a reduction of 6.0% the collector is set at yearly tilt angle.

4-It is important to record the solar spectrum during these four seasons period and you have to compare with daily spectrum during each day of winter, spring, autumn and summer with the calculated solar data of this paper.

5-This theoretical total yearly solar energy (1944 Kw.h.m<sup>2</sup>) is enough for heating water for average family in Kurdistan.

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### پوخته

ئیک ژ پارامیته‌رین گرنگ یین کو کارتیکرنی لسه‌ر کارئ کومکهرئ تیشکا روژئ دکهت نهو گوشه‌یه یا دنافه‌را کومکهری رویی زه‌فیی دا یه نه‌فه ژبه‌رکو گهورینا فی گوشئ کارتیکرنی لسه‌ر برئ تیشکا روژئ دکهت نهوئ دگه‌هیته سهر کومکهری. تیشکا روژئ یا هه موکی بو شارئ دهوکی کو ژ سالا 1990 تا کو سالا 2007 هاته بکار بو دهرئینانا باشترین گوشه بو کومکهرئ روژئ ل شارئ دهوکی. پشتی شیتله‌ل کرنا تیشکا روژئ یا هه موکی بوهرسی به‌شین وئ لسه‌ر کومکهری بو مه وه‌سا دیاربو باشترین گوشه دماوئ سالی‌دا دکه‌فیته دنافه‌را (0پله) بو مه‌ها خزیرانی و (62پله) بو مه‌ها کانینا دووئ. دودرزی زفستانید مه‌هین (کانینا ئیکی، کانینا دووئ، سباتئ) باشترین گوشه دبیته (58پله) بومه‌هین بهاری (ئادار، نیسان، گولان) باشترین گوشه دبیته (20پله) بومه‌هین هافینئ ( خزیران، ته‌باخ، تیرمه‌ه) باشترین گوشه دبیته (5پله) بومه‌هین پائیزی (تشرینائیکی، تشرینا دووئ، نه‌یلول) باشترین گوشه دبیته (46.33پله) تیکرایئ سالانه بوباشترین گوشه دبیته (32.33پله).